Exercise Course
NETWORK ALGORITHMS

Exercises 1

General notes:

- All exercises that ask for an algorithm also ask for a proof of its correctness (unless this is trivial) and running time.
- Solutions to exercises may be submitted in English or German.

We consider a model in which every processor may send at most one packet per round to a neighbouring processor in an (undirected) network. Processors may receive an unlimited number of packets per round. We refer to the task of sending from every processor $P_i$ one packet to another processor $P_{\pi(i)}$ as permutation routing.

Exercise 1.1: (5 points)
A network of processors $\{P_0, \ldots, P_{n-1}\}$ is a ring of size $n$ iff every processor $P_i$ is connected to processor $P_{(i+1) \mod n}$. Describe an algorithm for the permutation routing problem on a ring of size $n$ with constant buffer size efficiently.

Exercise 1.2: (5 points)
Assume you are given an algorithm $A$ that operates on a ring of $n$ processors. The algorithm is specified by a sequence of rounds each of which consists of a set of messages that are passed between adjacent processors.

You want to execute this algorithm on another processor network which, unfortunately, is not a ring, but an $n$-array $M(n,1)$. Simulate the behaviour of $A$ on the ring by simulating each round by no more than 4 rounds on the array.

Exercise 1.3: (5 points)
We now consider the broadcast problem, i.e., a packet is to be sent from a processor $P_0$ to all processors in the network. Describe algorithms that perform broadcasting on the following network topologies:

1. the grid $M(n,2)$ where $P_0$ is the “upper left” processor $(0,0)$ and
2. the complete binary tree of depth $n$ where $P_0$ is the root.

Exercise 1.4: (5 points)
Describe an algorithm for the permutation routing problem on the grid $M(n,2)$. Specify its running time and the required buffer size.

Deadline: Tuesday, April 18, 11:00, letterbox in front of i1.