Exercise 10.1: (5 points)
Consider a pure Nash equilibrium. Fix some parameter $k$ and let $q$ denote the leftmost (fastest) machine in $L_{k+1}$ and let $q'$ denote the leftmost machine in $L_{k-1}$. (Remember: We normalize $OPT = 1$.)

1. Prove: There exists a machine $j^* \in L_{k+2}$ and a job $i \in [n]$ such that job $i$ is assigned to machine $j^*$ and $w_i \leq s_{q'}$.
2. Prove $s_q \geq 2s_{q'}$.
   Hint: Use part 1 and the Nash property for machines $q'$ and $j^*$.

Exercise 10.2: (5 points)
Use Exercise 10.1.2 to prove an upper bound on the Price of Anarchy in terms of the ration between fastest and slowest speed $r = \frac{s_{\text{max}}}{s_{\text{min}}}$.

Exercise 10.3: (5 points)
Now consider a mixed Nash equilibrium. Fix $i \in [n]$, $j \in L_{k-1}$ such that $p_i^j \in (0, 1/4)$. Prove that $w_i \leq 12s_j$ in three steps:

1. Prove $c_i^j \geq k - 1 + \frac{2w_i}{s_j}$.
2. Prove $c_i^j \leq k + 2 + \frac{w_i}{s_q}$ with $q$ defined as in Exercise 10.1.
3. Exercise 10.1.2 implies that $s_q \geq 2s_j$ for the case of mixed equilibria also. Prove that this in conjunction with Exercise 10.3.1 and 10.3.2 implies $w_i \leq 12s_j$.

Deadline: Monday, July 3, 11:00, letterbox in front of i1.