Exercise 4.1: (5 points)
Consider an algorithm for the oblivious routing problem on $M(n,3)$ that uses paths that fix the individual dimensions subsequently, i.e., a packet from $a = (a_0, a_1, a_2)$ to $b = (b_0, b_1, b_2)$ is routed on a shortest path via $(b_0, a_1, a_2)$ and $(b_0, b_1, a_2)$.

What is the running time of a routing algorithm using these paths?

Exercise 4.2: (5 points)
Prove Remark 2.10 b) from the script:

Remark 2.10 b). Consider a network $M$ with $n$ nodes and degree $c$, and $m \leq n$ fixed source nodes and $m$ fixed sink nodes. Every oblivious routing protocol needs at least

$$\Omega \left( \frac{m}{\sqrt{c \cdot n}} \right)$$

steps in the worst case.

Exercise 4.3: (5 points)
For a positive constant $k$, consider the following random experiment: $k \cdot n$ balls are thrown uniformly at random into $n$ bins. Let $L_i$ be the random variable that describes the number of balls in bin $i$. Prove that $\max_{i} \{L_i\} = \mathcal{O}(k + \log n)$ with high probability, i.e., for every fixed constant $c$, there exists an $\alpha_c > 0$ such that

$$\Pr \left[ \max_{i \in [n]} L_i \leq c \cdot (k + \log n) \right] \geq 1 - n^{-\alpha_c}.$$