Exercise 8.1: (5 points)
Assume you are given a \((c, \delta)\)-power spanner \(G\) for a set of nodes \(V \subset \mathbb{R}^2\), i.e., for every pair of nodes \(v_0, v_\ell\) there exists a path \(p = (v_0, \ldots, v_\ell)\) such that \(||p||^\delta = \sum_{i=0}^{k-1} ||v_i v_{i+1}||^\delta \leq c \cdot ||v_0 v_\ell||^\delta\). This seems to be a very weak assertion since a minimum cost path \(q = (w_0, \ldots, w_k)\) in \(V\) between \(v_0\) and \(v_\ell\) (with \(w_0 = v_0\) and \(w_k = v_\ell\)) may have cost \(||q||^\delta\) that is much smaller than \(||v_0 v_\ell||^\delta\).
Prove that the above assertion implies the following stronger one:

\[ ||p||^\delta \leq c \cdot ||q||^\delta = c \cdot \sum_{i=0}^{k-1} ||w_i w_{i+1}||^\delta. \]

Exercise 8.2: (5 points)
The Yao graph is a power spanner and, hence, according to Exercise 8.1 contains power efficient paths between every pair of nodes. Unfortunately, the routing algorithm for the Yao graph (cf. lecture notes) does not necessarily find these paths. Let us construct a bad example.

For any \(n\), show how to construct a set of nodes \(V\) with \(|V| = n\) containing two nodes \(v, w \in V\) with the following property: Let \(p\) be the routing path from \(v\) to \(w\) in the Yao graph of \(V\). Let \(q\) be a power minimal path in \(V\).

\[ ||p||^\delta \geq ||q||^\delta \cdot \Omega(n^{\delta-1}). \]

Exercise 8.3: (5 points)

**Definition.** For any set of nodes \(V \subset \mathbb{R}^2\), the Delaunay graph \(\text{Del}(V)\) consists of all edges \((u, v)\) that have a node \(w \in V\) for which the smallest sphere containing \(u, v,\) and \(w\) does not contain any other node of \(V\).

Prove that the Delaunay graph contains all power minimal paths. Therefore, fix a power minimal path \(p\) in \(V \subset \mathbb{R}^2\).

1. For an edge \(e = (v, w)\) in \(p\), show that the smallest sphere containing \(v\) and \(w\) is empty.
2. Show that this implies that the Delaunay graph of \(V\) contains \(e\).

**Deadline:** Monday, June 19, 11:00, letterbox in front of i1.