Algorithmic Graph Theory (SS2016)

Chapter 3
Simple Intersection-Graphs

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Lehrstuhl für Informatik 1

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Basics

- A graph consists of nodes, which are “connected” by some relation.
- Often we have objects, for which some relation exists.
- Possible relations:
  - Objects have some common property.
  - Objects are neighbours.
  - Objects have some limited distance.
  - Objects intersect.
- We define intersection-graphs using the later relation.
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Definition

A graph $G = (V, E)$ is called intersection-graph of a set $\mathcal{M}$ of objects, iff $G = (V, E)$ is isomorphic to $H = (\mathcal{M}, \{\{a, b\} \mid a \cap b \neq \emptyset\})$. $\mathcal{M}$ is called the intersection representation of $G$.

Possible families of objects are:

- Intervals on a line.
- Arc of a circle.
- Chords of a circle.
- Circles in the plane.
- Parallelograms between two lines.
- And lots more.

By using different classes of object we get different graph classes.
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**Definition**

- A graph $G = (V, E)$ is $k$-colourable iff:
  - $\exists f : V \mapsto \{1, ..., k\} : \forall (a, b) \in E, f(a) \neq f(b)$.
- The function $f$ is called coloring of $G$.

**Definition**

- $\chi(G)$ is the chromatic number $\chi(G)$ of $G$, iff
- $G$ is $\chi(G)$-colourable, but is not $(\chi(G) - 1)$-colourable.
Colouring Problems

**Definition**

The graph-to-colour problem is the following:

**Input:** $G$ a graph

**Output:** Optimal colouring of $G$.

**Definition**

The colouring problem is the following:

**Input:** $k \in \mathbb{N}$ and a graph $G$

**Output:** Is $G$ $k$-colourable?

**Definition**

The $k$-colouring problem is the following:

**Input:** $G$ a Graph

**Output:** Is $G$ $k$-colourable?
Independent Set

Definition

- A graph $G = (V, E)$ contains an independent set of size $k$, iff
- $\exists S \subseteq V : |S| = k \land \forall a, b \in S, a \neq b : (a, b) \notin E$.

Definition

- $\alpha(G)$ denotes the size of the largest independent set:
- $G$ contains an independent set of size $\alpha(G)$, but no independent set of size $\alpha(G) + 1$. 
Definitions

Let $G = (V, E)$ be a graph.

\[ \alpha(G) = \max \{ |V'| : V' \subset V \land \forall a, b \in V' : (a, b) \notin E \} \]
\[ \omega(G) = \max \{ |V'| : V' \subset V \land \forall a, b \in V' : (a, b) \in E \} \]
\[ \chi(G) = \min \{ k : \exists V_1, V_2, \ldots, V_k : \bigcup_{i=1}^{k} V_i = V \land \]
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More notations:
\[ \omega(G) = \overline{\alpha}(G), \]
\[ \alpha(G) = \overline{\omega}(G) = \beta_0(G), \]
\[ \kappa(G) = \overline{\chi}(G) \]
First simple Example

- **Time of activity of a register (construction of a compiler)**
- **Program segments:** \( \cdots \text{Read}(A) \cdots \text{Write}(B) \cdots \)
- **Living time of a variable \( A \): Maximal interval**
  - Starting with a \( \text{Write}(A) \).
  - Ending by the last \( \text{Read}(A) \).
  - Such that no further \( \text{Write}(A) \) is between this two points.
- **Problem:** how many registers are needed?
- D.h. assign for each living time of a variable a register.
- **Example:** \((0, 10), (3, 7), (9, 20), (25, 50), (12, 34), (6, 16), (17, 26), (11, 46), (23, 26), (30, 46), (19, 27)\)
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**Interval-graphs**

**Definition (Interval-graphs)**

- A graph $G = (V, E)$ is called interval-graph, iff
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Model and Colouring (Idea)

Idea: look for independent sets.
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![Diagram showing independent sets in a model and colouring context.](image)
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Idea: check the intervalls from left to the right (sorted by the left endpoints):
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Determine the invariant:
Model and Colouring (Invariant)

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0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50

\(a\) \(b\) 
\(c\) 
\(d\) 
\(e\) 
\(f\) 
\(g\) 
\(h\) 
\(i\) 
\(j\) 
\(k\)
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Model and Colouring (Invariant)

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\begin{center}
\begin{figure}
\begin{tikzpicture}
\draw[very thin] (0,0) grid (51,1);
\foreach \x in {0,2,4,...,50}
{\draw[|-,thick] (\x,0.5) -- (\x,0.75) node[below, align=center] {\x};}
\foreach \y in {a,b,c,d,e,f,g,h,i,j,k}
{\draw[\y,thick] (2,1-0.5) -- (2+1,1-0.5) node[below, align=center]{\y};}
\end{tikzpicture}
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```

![Diagram showing model and colouring invariant](attachment:image.png)
Theorem

The graph-to-colour problem is for interval-graphs in time $O(n \log(n))$ solvable.

1. Sort the intervals by their left endpoints.
2. Check all endpoints $e$ from the left to the right.
3. If $e$ is the starting point of an interval, colour it with the smallest free colour.
4. If $e$ is the ending point of an interval $I$ is, free the colour of $I$.

Invariant

If a node $v$ is coloured with colour $k$, then $v$ is part of a $k$-clique.
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If a node $v$ is coloured with colour $k$, then $v$ is part of a $k$-clique.
Colouring of Interval-graphs (Algorithm)

**Theorem**

*The graph-to-colour problem is for interval-graphs in time $O(n \log(n))$ solvable.*

1. Sort the intervals by their left endpoints.
2. Check all endpoints $e$ from the left to the right.
3. If $e$ is the starting point of an interval, colour it with the smallest free colour.
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Example of independent set problem on intervall-graphs

1. Sort the intervals by their starting points.
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![Diagram showing an example of independent set problem on interval-graphs.](image)
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Independent Set Problem for Interval-graphs

**Theorem**

Finding a maximal independent set is solvable in time $O(n \log(n))$ on interval-graphs.

1. Sweep through the start- and endpoints of intervals from left to right.
2. Store for each endpoint $e$ the size of a maximal independent set of intervals, which is placed to the left of $e$.
3. While sweeping from left to right do:
   1. If $e$ is a starting point of interval $(e, f)$ and there is no endpoint to the left of $e$, then let $S(f) = 1$.
   2. If $e$ is a starting point of interval $(e, f)$, then compute: largest endpoint $e'$ to the left of $e$ and let $S(f) = S(e') + 1$.
   3. If $e$ is an endpoint of interval $(a, e)$, then compute: largest endpoint $e'$ to the left of $e$ and to the right of $a$. If that exists, then let $S(e) = \max(S(e'), S(e))$. 
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Maximal Clique on Interval-graphs

**Theorem**

*Finding a maximal clique is solvable in time* \(O(n \log(n))\) *on interval-graphs.*

**Remark**

Very many problems are efficient solvable on interval-graphs.
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Definition (Permutations-Graph)

A graph $G = (V, E)$ is called permutation-graph,
iff he is definable by a permutation $\pi : \{1..n\} \rightarrow \{1..n\}$ in the following way:
$G = (\{1..n\}, \{(i, j); (i - j)(\pi(i) - \pi(j)) < 0\})$.

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A permutation-graph is the intersection graph of a set of lines, which are drawn between to parallel lines.
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Example and Colouring

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Idea: Analog algorithms as on intervall-graphs.

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**Definition (Arc-Graph)**

- A graph $G = (V, E)$ is called arc-graph,
- iff he is the intersection graph of a set of arcs on a circle.
- A arc-graph is called **proper**, iff no arc in contained in an other arc.

**Remark**

An interval-graph is an arc-graph.

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Are the algorithms for interval-graphs adaptable to arc-graphs.
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- Question, what is the reason that the above problems are efficient solvable on intervall-graphs?
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- One could think, all $k!$ colourings should be considered (stored).
- But, the colourings are exchangeable.
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- One could think, all $k!$ colourings should be considered (stored).
- But, the colourings are exchangeable.
- Thus only the optimal colouring at each position is stored.

Question:
What is the situation on arc-graphs?
Reasoning for the above Results

- Question, what is the reason that the above problems are efficient solvable on interval-graphs?

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**Question:**

What is the situation on arc-graphs?
Consider the flow of information.

What information has to be considered when moving around the circle?

The colouring are not exchangeable because the end the colours have to match.

Thus we may have to consider $k!$ colourings.

If $k$ is constant, then the problem is in $\mathcal{P}$

IF $k$ is not constant, then the problem could be in $\mathcal{NPC}$. 
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![Diagram of arc-graphs with different colourings]
Theorem

The $k$-colouring problem on arc-graphs is solvable in polynomial time.

Idea: Consider all $k!$ colourings.

1. W.l.o.g.: The graph contains no $k + 1$ clique.
2. Otherwise we search analog as on interval-graphs for the largest clique.
3. Colour an some maximal $k'$-Clique.
4. Colour the arcs in a clockwise order.
5. At most $k!$ colourings are considered (stored) during this process.
6. Check at the end if some colouring do not contradict with the first one.
7. Running time: $O(k!^2 \cdot n \log n) = O(n \log n)$
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Colouring Problem on Arc-Graphs

**Theorem**

The colouring problem on arc-graphs NP-complete.

**Idea:** Reduction to the word problem for symmetric groups.

**Definition**

The word problem for symmetric groups is the following:

**Input:** \( \pi \in S_k \) (Word and symmetric group) and \( S_1, S_2, \cdots S_n \) subgroups

**Output:** Holds: \( \pi \in S_1 \circ S_2 \circ \cdots \circ S_n \)
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Colouring Problem on Arc-Graphs
Colouring Problem on Arc-Graphs

\[ S_1 = \{2, 4\} \]
Colouring Problem on Arc-Graphs

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Colouring Problem on Arc-Graphs

$S_1 = \{2, 4\}$
Colouring Problem on Arc-Graphs

\[ \pi(1) = 3 \]
\[ \pi(2) = 1 \]
\[ \pi(3) = 2 \]
\[ \pi(4) = 5 \]
\[ \pi(5) = 4 \]
\[ \pi(6) = 6 \]
Colouring Problem on Arc-Graphs

$S_1 = \{2, 4\}$

$S_2 = \{4, 6\}$
Colouring Problem on Arc-Graphs

\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
Colouring Problem on Arc-Graphs

$S_1 = \{2, 4\}$

$S_2 = \{4, 6\}$

$\pi(1) = 3$

$\pi(2) = 1$

$\pi(3) = 2$

$\pi(4) = 5$

$\pi(5) = 4$

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Colouring Problem on Arc-Graphs

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\[ S_2 = \{4, 6\} \]
Colouring Problem on Arc-Graphs

\[ \pi(1) = 3 \]
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\[ \pi(5) = 4 \]
\[ \pi(6) = 6 \]

- \( S_1 = \{2, 4\} \)
- \( S_2 = \{4, 6\} \)
- \( S_3 = \{1, 3\} \)
Colouring Problem on Arc-Graphs

$S_1 = \{2, 4\}$

$S_2 = \{4, 6\}$

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Colouring Problem on Arc-Graphs

$S_1 = \{2, 4\}$

$S_2 = \{4, 6\}$

$S_3 = \{1, 3\}$
Colouring Problem on Arc-Graphs

\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
\[ S_3 = \{1, 3\} \]
Colouring Problem on Arc-Graphs

\[ \pi(1) = 3, \quad \pi(2) = 1, \quad \pi(3) = 2, \quad \pi(4) = 5, \quad \pi(5) = 4, \quad \pi(6) = 6 \]

\[ S_1 = \{2, 4\}, \quad S_2 = \{4, 6\}, \quad S_3 = \{1, 3\} \]
Colouring Problem on Arc-Graphs

\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
\[ S_3 = \{1, 3\} \]
Colouring Problem on Arc-Graphs

\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
\[ S_3 = \{1, 3\} \]
Colouring Problem on Arc-Graphs

$S_1 = \{2, 4\}$

$S_2 = \{4, 6\}$

$S_3 = \{1, 3\}$

$S_4 = \{1, 6\}$
Colouring Problem on Arc-Graphs

$\pi(1) = 3$
$\pi(2) = 1$
$\pi(3) = 2$
$\pi(4) = 5$
$\pi(5) = 4$
$\pi(6) = 6$

$S_1 = \{2, 4\}$
$S_2 = \{4, 6\}$
$S_3 = \{1, 3\}$
$S_4 = \{1, 6\}$
Colouring Problem on Arc-Graphs

- $S_1 = \{2, 4\}$
- $S_2 = \{4, 6\}$
- $S_3 = \{1, 3\}$
- $S_4 = \{1, 6\}$
Colouring Problem on Arc-Graphs

\[ \pi(1) = 3 \]
\[ \pi(2) = 1 \]
\[ \pi(3) = 2 \]
\[ \pi(4) = 5 \]
\[ \pi(5) = 4 \]
\[ \pi(6) = 6 \]

\( S_1 = \{2, 4\} \)
\( S_2 = \{4, 6\} \)
\( S_3 = \{1, 3\} \)
\( S_4 = \{1, 6\} \)
Colouring Problem on Arc-Graphs

\[
\begin{align*}
S_1 & = \{2, 4\} \\
S_2 & = \{4, 6\} \\
S_3 & = \{1, 3\} \\
S_4 & = \{1, 6\}
\end{align*}
\]

\[
\begin{align*}
\pi(1) & = 3 \\
\pi(2) & = 1 \\
\pi(3) & = 2 \\
\pi(4) & = 5 \\
\pi(5) & = 4 \\
\pi(6) & = 6
\end{align*}
\]
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Colouring Problem on Arc-Graphs

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\[ \pi(2) = 1 \]

\[ \pi(3) = 2 \]

\[ \pi(4) = 5 \]

\[ \pi(5) = 4 \]

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Colouring Problem on Arc-Graphs

\[ \pi(1) = 3 \]

\[ S_1 = \{2, 4\} \]
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Colouring Problem on Arc-Graphs

\[ \pi(1) = 3 \]
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\[ S_2 = \{4, 6\} \]
\[ S_3 = \{1, 3\} \]
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Colouring Problem on Arc-Graphs

\[ \pi(2) = 1 \]
\[ \pi(1) = 3 \]

\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
\[ S_3 = \{1, 3\} \]
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Colouring Problem on Arc-Graphs

\[\begin{align*}
\pi(1) &= 3 \\
\pi(2) &= 1 \\
\pi(3) &= 2 \\
\pi(4) &= 5 \\
\pi(5) &= 4 \\
\pi(6) &= 6
\end{align*}\]

- \(S_1 = \{2, 4\}\)
- \(S_2 = \{4, 6\}\)
- \(S_3 = \{1, 3\}\)
- \(S_4 = \{1, 6\}\)
Colouring Problem on Arc-Graphs

\[ \pi(3) = 2 \]
\[ \pi(2) = 1 \]
\[ \pi(1) = 3 \]

\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
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Colouring Problem on Arc-Graphs

\[ \pi(4) = 5 \]
\[ \pi(3) = 2 \]
\[ \pi(2) = 1 \]
\[ \pi(1) = 3 \]

\( S_1 = \{2, 4\} \)
\( S_2 = \{4, 6\} \)
\( S_3 = \{1, 3\} \)
\( S_4 = \{1, 6\} \)
Colouring Problem on Arc-Graphs

\[ \pi(1) = 3 \]
\[ \pi(2) = 1 \]
\[ \pi(3) = 2 \]
\[ \pi(4) = 5 \]

\( S_1 = \{2, 4\} \)
\( S_2 = \{4, 6\} \)
\( S_3 = \{1, 3\} \)
\( S_4 = \{1, 6\} \)
Colouring Problem on Arc-Graphs

\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
\[ S_3 = \{1, 3\} \]
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Colouring Problem on Arc-Graphs

\[ \begin{align*}
\pi(1) &= 3 \\
\pi(2) &= 1 \\
\pi(3) &= 2 \\
\pi(4) &= 5 \\
\pi(5) &= 4 \\
\end{align*} \]

\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
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**Colouring Problem on Arc-Graphs**

\[
\begin{align*}
\pi(6) &= 6 \\
\pi(5) &= 4 \\
\pi(4) &= 5 \\
\pi(3) &= 2 \\
\pi(2) &= 1 \\
\pi(1) &= 3
\end{align*}
\]

\[
\begin{align*}
S_1 &= \{2, 4\} \\
S_2 &= \{4, 6\} \\
S_3 &= \{1, 3\} \\
S_4 &= \{1, 6\}
\end{align*}
\]
Colouring Problem on Arc-Graphs

\[
\begin{align*}
\pi(6) & = 6 \\
\pi(5) & = 4 \\
\pi(4) & = 5 \\
\pi(3) & = 2 \\
\pi(2) & = 1 \\
\pi(1) & = 3
\end{align*}
\]

\[
\begin{align*}
S_1 & = \{2,4\} \\
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S_3 & = \{1,3\} \\
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Colouring Problem on Arc-Graphs

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S_1 &= \{2, 4\} \\
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\end{align*} \]
Circle-Graphs

Definition (Circle-Graphs)

- A graph $G = (V, E)$ is called circle-graph,
- iff it is the intersection graph of a set of chords within one circle.

Definition (Overlap-Graph)

- A graph $G = (V, E)$ is called overlap-graph,
- iff it is definable by the overlapping of a set of intervals on a line.
- Let $I$ be a set of intervals.
- Then the corresponding overlap-graph is:
  $$G = (I, \{(a, b) \mid a, b \in I \land a \setminus b \neq \emptyset \land b \setminus a \neq \emptyset \land a \cap b \neq \emptyset\})$$
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Example
Example
Example
Example
Example
Example
Example
Example
Example
Example
Statements on Circle-Graphs

Lemma

1. An interval-graph is an arc-graph.
2. A proper arc-graph is a circle-graph.
3. A permutation-graph is a circle-graph.
4. A graph $G$ is a circle-graph, iff $G$ is a overlap-graph.

Just show: a graph $G$ is a circle-graph, iff $G$ is a overlap-graph.

- Chord $A$ from $r \cdot e^{i \cdot a}$ to $r \cdot e^{i \cdot a'}$ becomes interval $A' = (a, a')$ ($0 \leq a < a' < 2 \cdot \pi$).
- Chord $B$ from $r \cdot e^{i \cdot b}$ to $r \cdot e^{i \cdot b'}$ becomes interval $b' = (b, b')$ ($0 \leq b < b' < 2 \cdot \pi$).
- The chord $A$ crosses $B$, iff $a < b < a' < b'$ oder $b < a < b' < a'$.
- Interval $A'$ has an overlap with $B$, iff $a < b < a' < b'$ oder $b < a < b' < a'$. 
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- The chord $A$ crosses $B$, iff $a < b < a' < b'$ oder $b < a < b' < a'$.
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Colouring of Circle-Graphs (Idea)

- What is the flow of information?
- Crossing chords “limit” the flow of information.
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- Thus, the 4-colouring problem on circle-graphs could be NP-complete.
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**Theorem**

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The 3-colouring problem on circle-graphs is solvable in time $O(n \log(n))$. 
Colouring Problems (Overview)

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4-Colouring Problem on Circle-Graphs

- **Reduction from the 3-SAT Problem.**
- For a given 3-SAT formula $\mathcal{F}$ we construct a circle-graph $G$.
- It has to hold: $\mathcal{F}$ satisfiable $\iff$ $G$ 4-colourable.
- Problem: Coding of logical values by the colouring of cords.
- Idea: Each pair of chord $(a, b)$ codes a logical value of $\nu$.
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Construct some kind of “circuit”.
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Overview

\[ c_1^n \quad \cdots \quad c_m^n \]

\[ x_1^{0 \ldots n} \quad x_1^{1 \ldots n} \quad x_1^{2 \ldots n} \quad \cdots \quad x_{m-1}^{m-1 \ldots 1} \quad x_n^{n \ldots 1} \]

\[ x_1^{m \ldots 1} \quad \cdots \quad x_1^{n \ldots 1} \]

\[ x_1^{1 \ldots n} \]
Overview
The Negation

Negation II: $x = \neg y$

Combination of Colours

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Negation II: \( x = \neg y \)

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\hline
x & \rightarrow & a & \leftarrow & y \\
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![Diagram showing negation](image)

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Some Simple Components

Negation II:
\[ x = \neg y \]
Some Simple Components

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Negation II:  \[ x = \neg y \]

Equality:  \[ x = y \]
Some Simple Components

Negation II:  
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Some Simple Components

Negation II:
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Equality:
\[ x = y \]

Static XOR:
\[ x = y \oplus e \]
Some Simple Components

Negation II: \( x = \neg y \)

Equality: \( x = y \)

Static XOR: \( x = y \oplus e \)
Equality: \((x = y = z)\)

- \(\neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z\)
- \(\neg y \Rightarrow b_1 \Rightarrow \neg x\)
- \(y \Rightarrow \neg a_1 \Rightarrow \neg a_2 \Rightarrow z\)
- \(y \Rightarrow \neg a_2 \Rightarrow b_2 \Rightarrow \neg b_1 \Rightarrow x\)

A colouring is possible in all cases.
Equality: \((x = y = z)\)

- \(\neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z\)
- \(\neg y \Rightarrow b_1 \Rightarrow \neg x\)
- \(y \Rightarrow \neg a_1 \Rightarrow \neg a_2 \Rightarrow z\)
- \(y \Rightarrow \neg a_2 \Rightarrow b_2 \Rightarrow \neg b_1 \Rightarrow x\)

A colouring is possible in all cases.
Equality: $(x = y = z)$

- $\neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z$
- $\neg y \Rightarrow b_1 \Rightarrow \neg x$
- $y \Rightarrow \neg a_1 \Rightarrow \neg a_2 \Rightarrow z$
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- \(y \Rightarrow \neg a_1 \Rightarrow \neg a_2 \Rightarrow z\)
- \(y \Rightarrow \neg a_2 \Rightarrow b_2 \Rightarrow \neg b_1 \Rightarrow x\)

A colouring is possible in all cases.
Equality: \((x = y = z)\)

- \(-y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow -z\)
- \(-y \Rightarrow b_1 \Rightarrow -x\)
- \(y \Rightarrow -a_1 \Rightarrow -a_2 \Rightarrow z\)
- \(y \Rightarrow -a_2 \Rightarrow b_2 \Rightarrow -b_1 \Rightarrow x\)
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Equality: \((x = y = z)\)

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\neg y &\Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z \\
\neg y &\Rightarrow b_1 \Rightarrow \neg x \\
y &\Rightarrow \neg a_1 \Rightarrow \neg a_2 \Rightarrow z \\
y &\Rightarrow \neg a_2 \Rightarrow b_2 \Rightarrow \neg b_1 \Rightarrow x \\
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A colouring is possible in all cases.
Equality: \((x = x' \land y = y')\)
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\[
\begin{array}{cccccc}
  y & y_1 & y_2 & y_3 & y' \\
  1,1 & 1,2 & 1,1 & 1,2 & 1,1 \\
  1,1 & 1,2 & 1,1 & 1,2 & 2,2 \\
  1,1 & & & & \\
  1,1 & & & & \\
\end{array}
\]
Equality: \( (x = x' \land y = y') \)
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\[
\begin{array}{c|c|c|c|c}
  y & y_1 & y_2 & y_3 & y' \\
  \hline
  1,1 & 1,2 & 1,1 & 1,2 & 1,1 \\
  1,1 & 1,2 & 1,1 & 1,2 & 2,2 \\
  1,1 & 1,2 & 1,1 & 1,3 & 3,3 \\
  1,1 & 1,2 & 1,1 & 1,4 & 4,4 \\
  1,2 & 1,1 & 1,2 & 1,1 & 1,2 \\
  1,2 & 1,1 & 1,2 & 1,1 & 1,3 \\
  1,2 & 1,1 & 1,2 & 1,1 & 1,4 \\
  1,2 & 1,1 & 1,2 & 2,2 & 2,3 \\
  1,2 & 1,1 & 1,2 & 2,2 & 2,4 \\
  1,2 & & & & \\
\end{array}
\]
Equality: \((x = x' \land y = y')\)
Equality \((x = y = z)\)
Equality \( (x = y = z) \)

\[ x = y = z \]

\[ x = x' \text{ and } y = y' \]
Equality \((x = y = z)\)

- \(x = y = z\)

- \(x = x'\) and \(y = y'\)
Equality \((x = y = z)\)

\[ x = y = z \]

\[ x = x' \text{ and } y = y' \]

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More Simple Components

Weak Or:
\[ \neg x \land \neg z \Rightarrow \neg y \]
More Simple Components

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\( \neg x \land \neg z \Rightarrow \neg y \)
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Weak Negation:
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\[ \neg x \land \neg z \Rightarrow \neg y \]

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\[ \neg x \Rightarrow y \text{ and } \neg y \Rightarrow x \]

**True:**
\[ x = true \]
More Simple Components

Weak Or:
\[ \neg x \land \neg z \Rightarrow \neg y \]

Weak Negation:
\[ \neg x \Rightarrow y \text{ and } \neg y \Rightarrow x \]

True:
\[ x = true \]
More Simple Components

Weak Or:
\[ \neg x \land \neg z \Rightarrow \neg y \]

Weak Negation:
\[ \neg x \Rightarrow y \quad \text{and} \quad \neg y \Rightarrow x \]

True:
\[ x = true \]
Or \((x \lor y = z)\)

- \(x \land \neg y \Rightarrow \neg x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z\)
- \(x \Rightarrow \neg x' \Rightarrow z_1 \Rightarrow z\)
- \(y \Rightarrow \neg y' \Rightarrow z_4 \Rightarrow z\)
- A colouring is possible in all cases.
Or \((x \lor y = z)\)

\[\begin{align*}
\neg x \land \neg y & \Rightarrow \neg x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z \\
x & \Rightarrow \neg x' \Rightarrow z_1 \Rightarrow z \\
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A \text{ colouring is possible in all cases.}
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\[
x 
\quad \iff \quad \neg \neg \quad \iff \quad \neg \lor \\
\quad \iff \quad \neg \quad \iff \quad \neg \\
\neg x \land \neg y \Rightarrow \neg x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z
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- A colouring is possible in all cases.
Static Simple Clause

\[ x_2 = x'_2 \text{ and } (x_1 \oplus e_1) \lor (x_2 \oplus e_2) \lor (x_3 \oplus e_3) = true \]
Static Simple Clause

\[ x_2 = x'_2 \text{ and } (x_1 \oplus e_1) \lor (x_2 \oplus e_2) \lor (x_3 \oplus e_3) = true \]
Multiple Equality \((x_i = y_i)\)
Multiple Equality \((x_i = y_i)\)
Multiple Equality \((x_i = y_i)\) [with Transport \((z_0 = z_k)\)]
Multiple Equality \((x_i = y_i)\) [with Transport \((z_0 = z_k)\)]
Clause \((x_i = y_i \text{ und } c_i \text{ satisfied})\)
Clause \((x_i = y_i \text{ und } c_i \text{ satisfied})\)
Formula (all $c_i$ are satisfied)
Theorem

The $k$-colouring problem on circle-graphs is NP-complete for $k \geq 4$.

Theorem

The $(2 \cdot k - 1)$-colouring problem on circle-graphs with clique size $k$ is NP-complete for $k \geq 3$.

Theorem

A circle-graph with clique size $k$ is always $(3 \cdot k)$-colourable.
**Theorem**

Finding a maximal independent set is solvable in time $O(n \log(n))$ on circle-graphs.

**Theorem**

Finding a maximal clique is solvable in time $O(n \log(n))$ on circle-graphs.
**Theorem**

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Concluding Remarks

**Theorem**

*On an interval graph $G$ we may in time $O(n \log(n))$ compute $\chi(G)$, $\alpha(G)$ and $\omega(G)$.*

**Theorem**

*On a permutation graph $G$ we may in time $O(n \log(n))$ compute $\chi(G)$, $\alpha(G)$ and $\omega(G)$.*

**Theorem**

*The $k$-colouring problem on arc-graphs is solvable in polynomial time, but the colouring problem for arc-graphs is NP-complete.*

**Theorem**

*The 3-colouring on circle-graphs is solvable in time $O(n \log(n))$. The $k$-colouring problem on circle-graphs is NP-complete for $k \geq 4$.***
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The 3-colouring on circle-graphs is solvable in time $O(n \log(n))$. The $k$-colouring problem on circle-graphs is NP-complete for $k \geq 4$. 
Conclusions

- Colouring (and many more problems) on interval graphs are easy.
- \( k \)-colouring on arc-graphs is easy.
- Colouring on arc-graphs is hard.
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**Definition (g-Segment-graphs)**

A graph $G = (V, E)$ is called a g-Segment-graph, iff it is the intersection-graph of a set of chords within a regular $g$-polygon.

**Lemma**

We have:

1. A permutation-graph is a circle-graph.
2. A permutation-graph is a g-segment-graph.
3. A proper arc-graph is a circle-graph.
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Disk-graphs

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- A graph \( G = (V, E) \) is called disk-graph, iff
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Definition (Unit-Disk-graphs)

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## Overview of the Results

<table>
<thead>
<tr>
<th></th>
<th>$k$-Col.</th>
<th>Col.</th>
<th>Opt-Col</th>
<th>Ind.</th>
<th>Clique</th>
<th>Recognition</th>
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<td>Intervall</td>
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<td>Circle-G.</td>
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<td>Circle-G.</td>
<td>$\mathcal{NP}$</td>
<td>$\mathcal{NP}$</td>
<td>$\mathcal{NP}$-hard</td>
<td>$\mathcal{P}$</td>
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<td>$g$-Segment</td>
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<tr>
<td>Disk-G.</td>
<td>$\mathcal{NP}$</td>
<td>$\mathcal{NP}$</td>
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<tr>
<td>Planar</td>
<td>$\mathcal{NP}$</td>
<td>$\mathcal{NP}$</td>
<td>$\mathcal{NP}$-hard</td>
<td>$\mathcal{NP}$</td>
<td>$\mathcal{P}$</td>
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<tr>
<td>$k$-Planar</td>
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</tbody>
</table>
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- How is the colouring problem solvable on intervall graphs?
- How is the colouring problem solvable on permutation graphs?
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- Why is the colouring problem on arc-graphs hard?
- What is the idea of the reduction of the 4-colouring-problem on cycle-graphs?
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Legend

- : Not of relevance
- : implicitly used basics
- : idea of proof or algorithm
- : structure of proof or algorithm
- : Full knowledge