Exercise 1 (6 points)

Prove by giving an algorithm that for caterpillars of hair length 1, a 2-approximation to the number of wavelengths needed for routing along a set of given paths (i.e. \( w(G'_{\mathcal{R}}) \)) can be computed in polynomial time. (Again, each direction of an edge can simultaneously be used by at most one path of the same wavelength.) What changes could possibly make the algorithm’s output get closer to the optimum? How can the algorithm be extended for general caterpillars?

Consider the following communication model (slightly differing from that in the lecture):

In each time step, every node in the network can still send or receive one arbitrarily large message over one single edge. It is neither possible for a node to send/receive messages over different edges at the same time, nor can it both send and receive something at the same time.

In each time step, messages can now be sent along a collection of edge-disjoint paths in the graph instead of just a matching. On such a path, intermediate nodes will not see the message that is just passing them by.

Exercise 2 (2 points)

Give an optimal algorithm for (one-way) min-broadcast on a line graph and prove its time requirement.

Exercise 3 (4 points)

For a complete binary tree with \( n \) leaf nodes, give a (one-way) min-broadcast algorithm that will run in no more than \( \log n + 1 \) time steps.

Exercise 4 (4 points)

Can one derive a optimal (one-way) min-broadcast algorithm for general, connected graphs with \( 2^k \) nodes from the previous two exercises? If so, explain informally how this algorithm would run and why there can be no better one. If not, explain by giving an example why the strategies used above will fail.