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Basics

- A graph consists of nodes, which are “connected” by some relation.
- Often we have objects, for which some relation exists.
- Possible relations:
  - Objects have some common property.
  - Objects are neighbours.
  - Objects have some limited distance.
  - Objects intersect.
- We define intersection-graphs using the later relation.
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We define intersection-graphs using the later relation.
Definition

A graph $G = (V, E)$ is called intersection-graph of a set $\mathcal{M}$ of objects, iff $G = (V, E)$ is isomorphic to $H = (\mathcal{M}, \{\{a, b\} \mid a \cap b \neq \emptyset\})$. $\mathcal{M}$ is called the intersection representation of $G$.

Possible families of objects are:

- Intervals on a line.
- Arc of a circle.
- Chords of a circle.
- Circles in the plane.
- Parallelograms between two lines.
- And lots more.

By using different classes of object we get different graph classes.
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A graph $G = (V, E)$ is \textit{k-colourable} iff:

1. $\exists f : V \mapsto \{1, \ldots, k\} : \forall (a, b) \in E, f(a) \neq f(b)$.
2. The function $f$ is called \textit{colouring} of $G$.

**Definition**

$\chi(G)$ is the \textit{chromatic number} $\chi(G)$ of $G$, iff

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The graph-to-colour problem is the following:
**Input:** $G$ a graph
**Output:** Optimal colouring of $G$.

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The colouring problem is the following:
**Input:** $k \in \mathbb{N}$ and a graph $G$
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Definition

- A graph $G = (V, E)$ contains an independent set of size $k$, iff
  - $\exists S \subset V : |S| = k \land \forall a, b \in S, a \neq b : (a, b) \notin E$.

Definition

- $\alpha(G)$ denotes the size of the largest independent set:
  - $G$ contains an independent set of size $\alpha(G)$, but no independent set of size $\alpha(G) + 1$. 
Independent Set

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Let $G = (V, E)$ be a graph.

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\begin{align*}
\alpha(G) &= \max \{ |V'| \mid V' \subset V \land \forall a, b \in V' : (a, b) \notin E \} \\
\omega(G) &= \max \{ |V'| \mid V' \subset V \land \forall a, b \in V' : (a, b) \in E \} \\
\chi(G) &= \min \{ k \mid \exists V_1, V_2, \ldots, V_k : \bigcup_{i=1}^{k} V_i = V \land \\
&\quad \forall i : 1 \leq i \leq k : \forall a, b \in V_i : (a, b) \notin E \} \\
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More notations:
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Bounds

Lemma

- \( \chi(G) \geq \omega(G) \)
- \( \chi(G) \geq n/\alpha(G) \)
- \( \chi(G) = \max\{\chi(B) \mid B \text{ is a block in } G\} \)

Block is another name for 2-connected component.

Theorem

Let \( G = (V, E) \) be a graph with maximal node degree of \( d \) (\( d = \Delta(G) \)). Then \( \chi(G) \leq d + 1 \) holds.

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Let \( G = (V, E) \) be a graph. Then \( \chi(G) = 2 \) holds, iff \( G \) contains no cycles of odd length.
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Let \( G = (V, E) \) be a graph with maximal node degree of \( d \) \((d = \Delta(G))\). Then \( \chi(G) \leq d + 1 \) holds.

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Let \( G = (V, E) \) be a graph. Then \( \chi(G) = 2 \) holds, iff \( G \) contains no cycles of odd length.
**Bounds**

**Lemma**

- \( \chi(G) \geq \omega(G) \)
- \( \chi(G) \geq n/\alpha(G) \)
- \( \chi(G) = \max\{\chi(B) \mid B \text{ is a block in } G\} \)

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- **Time of activity of a register (construction of a compiler)**
- **Program segments:** \( \cdots \text{Read}(A) \cdots \text{Write}(B) \cdots \)
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Interval-graphs

Definition (Interval-graphs)

- A graph $G = (V, E)$ is called interval-graph, iff it is the intersection graph of a set of intervals on a line.
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Model and Colouring (Idea)

Idea: look for independent sets.
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0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50

- b
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![Diagram showing a model with colors and numbers representing the invariant.](image-url)
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The graph-to-colour problem is for interval-graphs in time $O(n \log(n))$ solvable.

1. Sort the intervals by their left endpoints.
2. Check all endpoints $e$ from the left to the right.
3. If $e$ is the starting point of an interval, colour it with the smallest free colour.
4. If $e$ is the ending point of an interval $I$ is, free the colour of $I$.

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If a node $v$ is coloured with colour $k$, then $v$ is part of a $k$-clique.
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Theorem

Finding a maximal independent set is solvable in time $O(n \log(n))$ on interval-graphs.

1. Sweep through the start- and endpoints of intervals from left to right.
2. Store for each endpoint $e$ the size of a maximal independent set of intervals, which is placed to the left of $e$.
3. While sweeping from left to right do:
   1. If $e$ is a starting point of interval $(e, f)$ and there is no endpoint to the left of $e$, then let $S(f) = 1$.
   2. If $e$ is a starting point of interval $(e, f)t$, then compute: largest endpoint $e'$ to the left of $e$ and let $S(f) = S(e') + 1$.
   3. If $e$ is an endpoint of interval $(a, e)$, then compute: largest endpoint $e'$ to the left of $e$ and to the right of $a$. If that exists, then let $S(e) = \max(S(e'), S(e))$. 
Independent Set Problem for Interval-graphs

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1. Sweep through the start- and endpoints of intervals from left to right.
2. Store for each endpoint $e$ the size of a maximal independent set of intervals, which is placed to the left of $e$.
3. While sweeping from left to right do:
   1. If $e$ is a starting point of interval $(e, f)$ and there is no endpoint to the left of $e$, then let $S(f) = 1$.
   2. If $e$ is a starting point of interval $(e, f)$, then compute: largest endpoint $e'$ to the left of $e$ and let $S(f) = S(e') + 1$.
   3. If $e$ is an endpoint of interval $(a, e)$, then compute: largest endpoint $e'$ to the left of $e$ and to the right of $a$. If that exists, then let $S(e) = \max(S(e'), S(e))$. 
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**Remark**

*Very many problems are efficient solvable on interval-graphs.*
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Permutation-Graphs

Definition (Permutations-Graph)

- A graph $G = (V, E)$ is called permutation-graph,
- iff he is definable by a permutation $\pi : \{1..n\} \rightarrow \{1..n\}$ in the following way:
- $G = (\{1..n\}, \{(i, j); (i - j)(\pi(i) - \pi(j)) < 0\})$.

Theorem

A permutation-graph is the intersection graph of a set of lines, which are drawn between to parallel lines.
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Example and Colouring

The invariant is the same as the one on interval-graphs.
Example and Colouring

\[ \pi(i) \quad \pi(h) \quad \pi(e) \quad \pi(c) \quad \pi(f) \quad \pi(a) \quad \pi(b) \quad \pi(d) \quad \pi(k) \quad \pi(g) \quad \pi(j) \]

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- A graph \( G = (V, E) \) is called arc-graph,
- iff he is the intersection graph of a set of arcs on a circle.
- A arc-graph is called proper, iff no arc in contained in an other arc.

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An interval-graph is an arc-graph.

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Reasoning for the above Results

Question, what is the reason that the above problems are efficient solvable on interval-graphs?

Consider the “flow of information”, i.e.:

Which information is used (stored) when the algorithms move from left to right.

One could think, all $k!$ colourings should be considered (stored).

But, the colourings are exchangeable.

Thus only the optimal colouring at each position is stored.

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What is the situation on arc-graphs?
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What information has to be considered when moving around the circle?

The colouring are not exchangeable because the end the colours have to match.

Thus we may have to consider $k!$ colourings.

If $k$ is constant, then the problem is in $\mathcal{P}$.

If $k$ is not constant, then the problem could be in $\mathcal{NPC}$. 
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Colouring Problem on Arc-Graphs

Theorem

The \( k \)-colouring problem on arc-graphs is solvable in polynomial time.

Idea: Consider all \( k! \) colourings.

1. W.l.o.g.: The graph contains no \( k + 1 \) clique.
2. Otherwise we search analog as on interval-graphs for the largest clique.
3. Colour an some maximal \( k' \)-Clique.
4. Colour the arcs in a clockwise order.
5. At most \( k! \) colourings are considered (stored) during this process.
6. Check at the end if some colouring do not contradict with the first one.
7. Running time: \( O(k!^2 \cdot n \log n) = O(n \log n) \)
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Colouring Problem on Arc-Graphs

Theorem

The colouring problem on arc-graphs NP-complete.

Idea: Reduction to the word problem for symmetric groups.

Definition

The word problem for symmetric groups is the following:
Input: $\pi \in S_k$ (Word and symmetric group) and $S_1, S_2, \cdots S_n$ subgroups
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\[\pi(1) = 3, \pi(2) = 1, \pi(3) = 2, \pi(4) = 5, \pi(5) = 4, \pi(6) = 6\]
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\[ \begin{align*}
S_1 &= \{2, 4\} \\
S_2 &= \{4, 6\} \\
S_3 &= \{1, 3\}
\end{align*} \]
Colouring Problem on Arc-Graphs

\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
\[ S_3 = \{1, 3\} \]
\[ S_4 = \{1, 6\} \]
Colouring Problem on Arc-Graphs

\[ S_1 = \{2, 4\} \]
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\[ S_2 = \{4, 6\} \]
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Colouring Problem on Arc-Graphs

$\pi(1) = 3$

$S_1 = \{2, 4\}$

$S_2 = \{4, 6\}$

$S_3 = \{1, 3\}$

$S_4 = \{1, 6\}$
Colouring Problem on Arc-Graphs

\[ \pi(1) = 3 \]

- \( S_1 = \{2, 4\} \)
- \( S_2 = \{4, 6\} \)
- \( S_3 = \{1, 3\} \)
- \( S_4 = \{1, 6\} \)
Colouring Problem on Arc-Graphs

\[ \pi(2) = 1 \]
\[ \pi(1) = 3 \]

- \( S_1 = \{2, 4\} \)
- \( S_2 = \{4, 6\} \)
- \( S_3 = \{1, 3\} \)
- \( S_4 = \{1, 6\} \)
Colouring Problem on Arc-Graphs

\[
\begin{align*}
\pi(1) &= 3 \\
\pi(2) &= 1 \\
S_1 &= \{2, 4\} \\
S_2 &= \{4, 6\} \\
S_3 &= \{1, 3\} \\
S_4 &= \{1, 6\}
\end{align*}
\]
**Colouring Problem on Arc-Graphs**

\[ \pi(3) = 2 \]
\[ \pi(2) = 1 \]
\[ \pi(1) = 3 \]

- \( S_1 = \{2, 4\} \)
- \( S_2 = \{4, 6\} \)
- \( S_3 = \{1, 3\} \)
- \( S_4 = \{1, 6\} \)
Colouring Problem on Arc-Graphs

\[ \pi(1) = 3 \]
\[ \pi(2) = 1 \]
\[ \pi(3) = 2 \]

\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
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\[ S_4 = \{1, 6\} \]
Colouring Problem on Arc-Graphs

\[ \pi(1) = 3 \]
\[ \pi(2) = 1 \]
\[ \pi(3) = 2 \]
\[ \pi(4) = 5 \]

\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
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Colouring Problem on Arc-Graphs

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\[ \pi(1) = 3 \]
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\begin{align*}
\pi(1) &= 3 \\
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Circle-Graphs

**Definition (Circle-Graphs)**

- A graph $G = (V, E)$ is called circle-graph,
- iff it is the intersection graph of a set of chords within one circle.

**Definition (Overlap-Graph)**

- A graph $G = (V, E)$ is called overlap-graph,
- iff it is definable by the overlapping of a set of intervals on a line.
- Let $I$ be a set of intervals.
- Then the corresponding overlap-graph is:
  
  $$G = (I, \{(a, b) \mid a, b \in I \land a \setminus b \neq \emptyset \land b \setminus a \neq \emptyset \land a \cap b \neq \emptyset\})$$
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Statements on Circle-Graphs

Lemma

1. An interval-graph is an arc-graph.
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Just show: a graph $G$ is a circle-graph, iff $G$ is a overlap-graph.

- Chord $A$ from $r \cdot e^{i \cdot a}$ to $r \cdot e^{i \cdot a'}$ becomes interval $A' = (a, a')$ ($0 \leq a < a' < 2 \cdot \pi$).
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- What is the flow of information?
- Crossing chords “limit” the flow of information.
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- Thus, the 4-colouring problem on circle-graphs could be NP-complete. 
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Theorem

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Colouring Problems (Overview)
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- Reduction from the 3-SAT Problem.
- For a given 3-SAT formula $\mathcal{F}$ we construct a circle-graph $G$.
- It has to hold: $\mathcal{F}$ satisfiable $\iff$ $G$ 4-colourable.
- Problem: Coding of logical values by the colouring of cords.
- Idea: Each pair of chord $(a, b)$ codes a logical value of $v$.
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Idea: Each pair of chord \((a, b)\) codes a logical value of \( v \).
Holding: \( v \iff f(a) = f(b) \) for a colouring \( f \).
Construct some kind of “circuit”.
4-Colouring Problem on Circle-Graphs

- Reduction from the 3-SAT Problem.
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Component Negation I \( (x = \neg y) \)
Component Negation I ($x = \neg y$)
Component Negation I \((x = \neg y)\)
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Component Negation I \((x = \neg y)\)
The Negation

Negation II: $x = \neg y$

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<tr>
<th>Combination of Colours</th>
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<tbody>
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Some Simple Components

Negation II:
\[ x = \neg y \]
Some Simple Components

Negation II:
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Some Simple Components

Negation II:
\[ x = \neg y \]

Equality:
\[ x = y \]
Some Simple Components

Negation II:
\[ x = \neg y \]

Equality:
\[ x = y \]
Some Simple Components

Negation II:  
\[ x = \neg y \]

Equality:  
\[ x = y \]

Static XOR:  
\[ x = y \oplus e \]
Some Simple Components

Negation II:
\[ x = \neg y \]

Equality:
\[ x = y \]

Static XOR:
\[ x = y \oplus e \]
Equality: \((x = y = z)\)

\[\begin{align*}
\neg y &\Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z \\
\neg y &\Rightarrow b_1 \Rightarrow \neg x \\
y &\Rightarrow \neg a_1 \Rightarrow \neg a_2 \Rightarrow z \\
y &\Rightarrow \neg a_2 \Rightarrow b_2 \Rightarrow \neg b_1 \Rightarrow x
\end{align*}\]

A colouring is possible in all cases.
Equality: \((x = y = z)\)

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\begin{align*}
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Equality: \((x = y = z)\)

- \(\neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z\)
- \(\neg y \Rightarrow b_1 \Rightarrow \neg x\)
- \(y \Rightarrow \neg a_1 \Rightarrow \neg a_2 \Rightarrow z\)
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Equality: \((x = y = z)\)

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Equality: $(x = y = z)$

- $\neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z$
- $\neg y \Rightarrow b_1 \Rightarrow \neg x$
- $y \Rightarrow \neg a_1 \Rightarrow \neg a_2 \Rightarrow z$
- $y \Rightarrow \neg a_2 \Rightarrow b_2 \Rightarrow \neg b_1 \Rightarrow x$
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Equality: \((x = y = z)\)

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- \(y \Rightarrow \neg a_2 \Rightarrow b_2 \Rightarrow \neg b_1 \Rightarrow x\)

A colouring is possible in all cases.
Equality: \((x = x' \land y = y')\)
Equality: \((x = x' \land y = y')\)

\[
\begin{array}{c|c|c|c|c|c}
  y & y_1 & y_2 & y_3 & y' \\
  \hline
  1,1 & 1,2 & 1,1 & 1,2 & 1,1 \\
  1,1 &   &   &   &   \\
  1,1 &   &   &   &   \\
  1,1 &   &   &   &   \\
\end{array}
\]
Equality: \((x = x' \land y = y')\)
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\[
\begin{array}{cccccc}
   y & y_1 & y_2 & y_3 & y' \\
   1,1 & 1,2 & 1,1 & 1,2 & 1,1 \\
   1,1 & 1,2 & 1,1 & 1,2 & 2,2 \\
   1,1 & 1,2 & 1,1 & 1,3 & 3,3 \\
   1,1 & 1,2 & 1,1 & 1,4 & 4,4 \\
   1,2 & 1,1 & 1,2 & 1,1 & 1,2 \\
   1,2 & 1,1 & 1,2 & 1,1 & 1,3 \\
   1,2 & 1,1 & 1,2 & 1,1 & 1,4 \\
   1,2 & & & & \\
   1,2 & & & & \\
   1,2 & & & & \\
\end{array}
\]
Equality: \((x = x' \land y = y')\)
Equality: \((x = x' \land y = y')\)

```
\[
\begin{array}{cccc}
  y & y_1 & y_2 & y_3 \\
  1,1 & 1,2 & 1,1 & 1,2 \\
  1,1 & 1,2 & 1,1 & 1,2 \\
  1,1 & 1,2 & 1,1 & 1,3 \\
  1,1 & 1,2 & 1,1 & 1,4 \\
  1,2 & 1,1 & 1,2 & 1,1 \\
  1,2 & 1,1 & 1,2 & 1,1 \\
  1,2 & 1,1 & 1,2 & 1,1 \\
  1,2 & 1,1 & 1,2 & 2,2 \\
  1,2 & 1,1 & 1,2 & 2,2 \\
  1,2 & 1,1 & 1,3 & 3,3 \\
\end{array}
\]
```
Equality \((x = y = z)\)
Equality \((x = y = z)\)

\[ x = y = z \]

\[ x = x' \text{ and } y = y' \]
Equality ($x = y = z$)

$x = y = z$

$x = x'$ and $y = y'$
Equality \((x = y = z)\)
More Simple Components

Weak Or:

\[ \neg x \land \neg z \Rightarrow \neg y \]
More Simple Components

Weak Or:
\(-x \land \neg z \Rightarrow \neg y\)
More Simple Components

Weak Or:
\[ \neg x \land \neg z \Rightarrow \neg y \]

Weak Negation:
\[ \neg x \Rightarrow y \text{ and } \neg y \Rightarrow x \]
More Simple Components

Weak Or:
\[ \neg x \land \neg z \Rightarrow \neg y \]

Weak Negation:
\[ \neg x \Rightarrow y \text{ and } \neg y \Rightarrow x \]
More Simple Components

Weak Or:
\[ \neg x \land \neg z \Rightarrow \neg y \]

Weak Negation:
\[ \neg x \Rightarrow y \text{ and } \neg y \Rightarrow x \]

True:
\[ x = true \]
More Simple Components

Weak Or:
\[ \neg x \land \neg z \Rightarrow \neg y \]

Weak Negation:
\[ \neg x \Rightarrow y \quad \text{and} \quad \neg y \Rightarrow x \]

True:
\[ x = \text{true} \]
More Simple Components

Weak Or:
\[ \neg x \land \neg z \Rightarrow \neg y \]

Weak Negation:
\[ \neg x \Rightarrow y \land \neg y \Rightarrow x \]

True:
\[ x = true \]
Or \((x \lor y = z)\)

\[\neg x \land \neg y \Rightarrow \neg x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z\]

\[x \Rightarrow \neg x' \Rightarrow z_1 \Rightarrow z\]

\[y \Rightarrow \neg y' \Rightarrow z_4 \Rightarrow z\]

A colouring is possible in all cases.
Or \((x \lor y = z)\)

\[\neg x \land \neg y \Rightarrow \neg x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z\]

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A colouring is possible in all cases.
Or \((x \lor y = z)\)

- \(\neg x \land \neg y \Rightarrow \neg x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z\)
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A colouring is possible in all cases.
Or \((x \lor y = z)\)

- \(\neg x \land \neg y \Rightarrow \neg x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z\)
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- A colouring is possible in all cases.
Or $(x \lor y = z)$

- $\neg x \land \neg y \Rightarrow \neg x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z$
- $x \Rightarrow \neg x' \Rightarrow z_1 \Rightarrow z$
- $y \Rightarrow \neg y' \Rightarrow z_4 \Rightarrow z$
- A colouring is possible in all cases.
Or \((x \lor y = z)\)

\[
\begin{align*}
\lnot x \land \lnot y & \Rightarrow \lnot x_3 \land \lnot y_1 \Rightarrow \lnot z_3 \Rightarrow \lnot z \\
x & \Rightarrow \lnot x' \Rightarrow z_1 \Rightarrow z \\
y & \Rightarrow \lnot y' \Rightarrow z_4 \Rightarrow z
\end{align*}
\]

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\[
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y & \Rightarrow \neg y' \Rightarrow \neg z_4 \Rightarrow z \\
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A colouring is possible in all cases.
Static Simple Clause

\[ x_2 = x'_2 \text{ and } (x_1 \oplus e_1) \lor (x_2 \oplus e_2) \lor (x_3 \oplus e_3) = true \]
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Formula (all $c_i$ are satisfied)
Colouring problems

Theorem

The $k$-colouring problem on circle-graphs is NP-complete for $k \geq 4$.

Theorem

The $(2 \cdot k - 1)$-colouring problem on circle-graphs with clique size $k$ is NP-complete for $k \geq 3$.

Theorem

A circle-graph with clique size $k$ is always $(3 \cdot k)$-colourable.
Theorem

Finding a maximal independent set is solvable in time $O(n \log(n))$ on circle-graphs.

Theorem

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On an interval graph $G$ we may in time $O(n \log(n))$ compute $\chi(G)$, $\alpha(G)$ and $\omega(G)$.

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The $k$-colouring problem on arc-graphs is solvable in polynomial time, but the colouring problem for arc-graphs is NP-complete.

Theorem

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*On an interval graph G we may in time $O(n \log(n))$ compute $\chi(G)$, $\alpha(G)$ and $\omega(G)$.***

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- Colouring (and many more problems) on interval graphs are easy.
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**g-Segment-graphs**

**Definition (g-Segment-graphs)**

- A graph $G = (V, E)$ is called a $g$-Segment-graph, iff
- it is the intersection-graph of a set of chords within a regular $g$-polygon.

**Lemma**

We have:

1. A permutation-graph is a circle-graph.
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Legend

■ : Not of relevance
■ : implicitly used basics
■ : idea of proof or algorithm
■ : structure of proof or algorithm
■ : Full knowledge