Exercise 1 (1+1 points)

Characterize all trees over the vertex set \(\{1, 2, \ldots, n\}\)

(a) whose Prüfer Code consists of \((n - 2)\)-times the same number
(b) whose Prüfer Code consists of \(n - 2\) pairwise distinct numbers

Exercise 2 (4 points)

The graph \(K_n - e\) (speak: \(K_n\) minus an edge) results from the complete graph \(K_n\) on \(n \geq 3\) vertices by removing one edge. Determine the number of spanning trees in \(K_n - e\).

Exercise 3 (4 points)

Prove: For every integer \(q \geq 3\), there exists a graph \(G_q\) on \(2q - 1\) vertices that has exactly \(q^2\) spanning trees.

Exercise 4 (4 points)

Use the matrix-tree-theorem to determine the number of spanning trees in the complete bipartite graph \(K_{r,s}\) with \(r, s \geq 1\).

Hint: Remember the characteristic polynomial from your linear algebra class.