Exercise Sheet 6


- You have eight days to create a solution and it must be done in a group of two or three students.
- Write the name and enrollment number of each group member on every sheet that you hand in.
- To achieve the permission for the exam you must earn 50% of the sum of all points and present one of your solutions at least once.
- You can earn 50% bonus points by presenting your solution. At the beginning of every exercise session, you can mark the exercises that you want to present.
- If a student is not able to present a correct solution although he/she marked the exercise as presentable, he/she will lose all of his/her points on the exercise sheet.

Write for every solution the number of used Processors, the size of the memory, the used PRAM model and the running time of your algorithm.

Exercise 1 (2 points)
Give a parallel algorithm that calculates for all vertices the shortest paths to every other vertex. You should achieve a running time of $O(\log^2(n))$ with $O(n^3)$ processors.

Exercise 2 (4 points)
Assume you have a planar graph and want to color it in parallel with 6 colors. Use the algorithm from Exercise 1 and try to split the planar graph into two outer planar graphs to find a parallel algorithm that solves the task.

Exercise 3 (1 points)
Consider a cycle where the vertices have no ids but the orientation of the edges is known. This means that a vertex can differentiate its right and left edge. Moreover the communication per round is strongly limited. A vertex can send only one bit per round.

Devise a parallel algorithm that, given a maximal (not maximum) independent set, finds a 3-coloring in $O(1)$ using $n$ processors.

Exercise 4 (4 points)
Consider a cycle where the vertices have no ids and the orientation of the edges is unknown. This means that the edges can not be distinguished and it is not possible to learn the orientation. Moreover the communication per round is strongly limited. A vertex can send only one bit per round.

- a) Explain in one or two sentences why it is impossible to deterministically 3-color the vertices even given a maximal (not maximum) independent set.
- b) Devise a randomized parallel algorithm that, given a maximal independent set, finds a 3-coloring in $O(\log n)$ rounds with probability at least $1 - \frac{1}{n}$ using $n$ processors.