Exercise Sheet 1

Exercise 1 (4 points)
Let $A$ be an $r \times n$ Latin rectangle as defined on the slides. Show by using Hall’s theorem that $A$ can be enlarged to an $n \times n$ Latin square.

Exercise 2 (4 points)
Prove or disprove the following statements:

- $G = (V, E)$ is planar and hamiltonian $\Rightarrow \exists E_1, E_2 : E_1 \cup E_2 = E$, and for $i ∈ \{1, 2\}, G_i = (V, E_i)$ is outerplanar.
- The opposite direction also holds.

Exercise 3 (4 points)
Have a look at the new slides 2:20 and 2:21 online, on which the 5-colouring proof for planar graphs is applied to the case of a 4-colouring. Why is this not a correct proof that all planar graphs can be coloured with four colours?