Exercise 1 (4 points)

Prove the theorem from the lecture stating that the bandwidth of a graph is lower bounded by its diameter in the following way:

\[ bw(G) \geq \left\lceil \frac{n - 1}{diam(G)} \right\rceil \]

Exercise 2 (4 points)

What is the bandwidth of a \( m \times n \) grid graph? Give a labeling that achieves this bandwidth.

Exercise 3 (4 points)

Consider a \( m \times n \) torus graph, i.e. a grid graph with additional edges between the leftmost and rightmost node of each row, and also between the top and bottom node of each column. What is the bandwidth of such a graph? Again, also give a labeling with which this can be achieved.