ONLINE ALGORITHMS

Exercise Sheet 9

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• Exercises appear at the i1 homepage (http://algo.rwth-aachen.de/en/Lehre/SS17/Online.php) on Friday.
• You have seven days to create a solution and it must be done in a group of two or three students.
• Write the name, group number and enrollment number of each group member on every sheet that you hand in.
• To achieve the permission for the exam you must earn 50% of the sum of all points and present one of your solutions at least once.
• You can earn 50% bonus points by presenting your solution. At the beginning of every exercise session, you can mark the exercises that you want to present.
• If a student is not able to present a correct solution although he/she marked the exercise as presentable, he/she will lose all of his/her points on the exercise sheet.

Exercise 1 (3 points)
Knapsack Problem:
Suppose we consider an adversary that is not allowed to end the input after at most two requests, but that is required to give an arbitrarily long sequence of objects, where the weights of every object must be positive. More precisely, it must construct an input of length \( n \), for any \( n \in \mathbb{N}^+ \). What is the best lower bound you can prove?

Exercise 2 (3 points)
Consider the following variant of the knapsack problem, which we call the simple removable knapsack problem. Here, an online algorithm is allowed to remove objects it already packed into the knapsack. However, once an object is removed, it cannot be packed again. The algorithm possibly removes more than one object at one time step.

Prove that any deterministic online algorithm for the simple removable knapsack problem has a strict competitive ratio of at least \( \frac{3}{2} \).

Exercise 3 (8 points)
Please recall the cow exercise from Sheet 8.

The cow reaches the long fence but does not see any other cow, just \( k \) cats are sitting around the fence. The cats tell the cow that the other cows are late and will arrive at an uncertain time at the long fence. As a cow, she does the obvious: The cow asks the cats for help. One cat should sit in front of every gate, remember which cow went through the gate, ask the cow when and where it arrived at the fence and tell this to every cow that arrives at the gate.

The cats are a little bit confused but agree to help the cow, after all the cow helped to solve the sleeping place problem.

a) What will be the new strategy of a cow that arrives at a gate which was already used by another cow? Remember, the cows that arrive at a used gate gain additional information and are very smart.

b) Discuss the relation between the sum of walked distances and the sum of all optimal ways?