• Write the name, group number and enrollment number of each group member on every sheet that you hand in.

• To achieve the permission for the exam you must earn 50% of the sum of all points and present one of your solutions at least once.

Exercise 1 (2 points)
(a) Prove that flush when full (FWF) is also a marking algorithm.
(b) Prove that FIFO is \( k \)-competitive but its not a marking algorithm.

Exercise 2 (4 points)
Think of the following modification of the paging problem, we call it paging on the line. There are \( m \) different pages and after the page \( p_i \) was requested, the next possible page is either \( p_i - 1 \), \( p_i \) or \( p_i + 1 \). The case \( i \in \{0, m - 1\} \) is special because the adversary can only ask for \( p_0 \) and \( p_1 \) or \( p_m - 1 \) and \( p_m - 2 \).

(a) Analyze FIFO and LRU on this structure.
(b) How does the analysis change if we change the path structure to a cyclic structure, e.g., after the page \( p_0 \) was requested, the adversary can request \( p_0 \), \( p_1 \) or \( p_m - 1 \) and if \( p_m - 1 \) was requested, he can ask for \( p_0 \), \( p_m - 2 \), or \( p_m - 1 \).

Hint: Choose \( m \) dependent on the cache size \( k \).

Exercise 3 (4 points)
Think of the following modification of the paging problem, we call it 2-List-paging. The adversary has two Lists \( L_1 \) and \( L_2 \) of pages and corresponding pointers \( l_1 \) and \( l_2 \). One is sorted in ascending order and the other one in descending order. Thus \( L_1 = [p_{k+1}, \ldots, p_m, p_1, \ldots, p_k] \) and \( L_2 = [p_m, \ldots, p_1] \). The adversary can ask for a page that are on the pointer positions. They are initialized with 0 and increase if the corresponding List is used. So, in the first step, the adversary can only ask for \( p_{k+1} \) or \( p_m \). If \( p_{k+1} \) is requested, \( l_1 \) is increased by one and the adversary can ask for page \( p_{k+2} \) or \( p_m \). If a pointer reaches the end of the List, it is set to 0 and starts from the beginning.

Choose the number of different pages \( m = 3k \) and the number of requests \( n = 2m \).

(a) Show that LRU has a competitive ratio of \( \frac{3}{2} \).
(b) Show that one of the algorithms presented in the lecture has an improved competitive ratio if the adversary is restricted by the 2-List-paging setting.

Exercise 4 (4 points)
A hungry cow stands in front of the farmers fence. She knows that if she follows the fence in one direction she will reach the barn where she can eat. But she does not know the correct direction.

As a cow, she does the obvious: she walks one cow meter to one direction, then back and 2 cow meters into the other direction, then back and so on. Thus, the cow walks in step \( i \in \{0 \ldots \} 2^i \) steps into one direction measured from the start. Prove that this approach cannot have a competitive ratio smaller than 9.

Bonus presentation exercises: Write your tutor (fischer@algo.rwth-aachen.de or tarik.viehmann@rwth-aachen.de) a mail and announce that you would like to present a presentation exercise. For every exercise group, only one student is allowed to present an exercise. So, write in your mail which exercise you would like to present and your group number. You are allowed to use the whiteboard and the slides from the lecture.

Bonus Exercise 5 (4 points (bonus))
File-Allocation: upper and lower bound.

Slides: 1:33 to 1:46 (Handout)
**Bonus Exercise 6**

Paging: lower bound, upper bound for marking algorithms, lookahead and resource augmentation.

Slides: 1:68 bis 1:83 (Handout)

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**Deadline:** The solutions are to be handed in until **May 8, 11:45**, in the lecture or at the drop boxes at the Chair i1.