• Write the name, group number and enrollment number of each group member on every sheet that you hand in.

• To achieve the permission for the exam you must earn 50% of the sum of all points and present one of your solutions at least once.

Exercise 1
Consider the following randomized $k$-server algorithm that solves the problem on a circle:

$CIRC$: The online algorithm chooses randomly and uniformly a point $P$ on the circumference of the circle. Think about the point $P$, which remains unknown to the oblivious adversary, as a roadblock that breaks the circle into a line segment. On this line segment the online algorithm behaves like the $DCOV$ algorithm from the lecture.

Prove that $CIRC$ is $2k$ competitive.

Exercise 2
Since the sleeping place incident, the cow and the cats became friends. Therefore, the cow wanted to learn the name of the cats and asked a cat for her name. The cat answered $0^n$. The cow was a little bit confused and asked the next cat for her name. After a while, the cow understood, that there are $2^n$ cats, each with a unique name from $\{0, 1\}^n$.

(a) The cow knows that the cats are offended if more than $i$ letters are wrong. Only if $i$ or less letters are wrong, the cats are not offended. As a lazy cow, she does the obvious: She tries to remember as few names as possible. Thus, she abuses the kindness of the cats and wants to call as many cats as possible with the same name. How many names does she need to remember and why?

Assume $i = 1$ and $n = 2^y - 1$ for arbitrary $y > 1$.

(b) Derive a formula that describes a lower bound for the number of names that the cow needs to remember for arbitrary large $i$ and arbitrary $n$.

Exercise 3
The online independent set problem on paths asks for the largest set of vertices such that no two vertices of the set are adjacent. The adversary chooses a permutation $\pi : [1, \ldots, n] \rightarrow [1, \ldots, n]$ in which the vertices $v_1, \ldots, v_n$ of the path are revealed to the algorithm. In time step $i$, the algorithm sees the vertex $v_{\pi(i)}$ with adjacent edges to already presented vertices.

(a) Is the problem competitive? Prove your answer.

(b) Derive an upper and lower bound for the advice complexity to become optimal.

Exercise 4
Knapsack Problem:

Suppose we consider an adversary that is not allowed to end the input after at most two requests, but that is required to give an arbitrarily long sequence of objects, where the weights of every object must be positive. More precisely, it must construct an input of length $n$, for any $n \in \mathbb{N}^+$. What is the best lower bound you can prove?

Bonus presentation exercises: Write your tutor (fischer@cs.rwth-aachen.de or tarik.viehmann@rwth-aachen.de) a mail and announce that you would like to present a presentation exercise. For every exercise group, only one student is allowed to present an exercise. So, write in your mail which exercise you would like to present and your group number. You are allowed to use the whiteboard and the slides from the lecture.

Bonus Exercise 5
Lower Bound for the $k$-server problem with advice.

Slides: 6:2 to 6:10 (Handout)
Bonus Exercise 6

Lower Bound for the $k$-server problem with advice (general case).

Slides: 6:11 to 6:21 (Handout)