Exercise 1
Consider the following variant of the knapsack problem, which we call the **simple removable knapsack problem**. Here, an online algorithm is allowed to remove objects it already packed into the knapsack. However, once an object is removed, it cannot be packed again. The algorithm possibly removes more than one object at one time step.

Prove that any deterministic online algorithm for the simple removable knapsack problem has a strict competitive ratio of at least $\frac{3}{2}$.

Exercise 2
The vertex cover problem asks for a graph $G = (V, E)$ for a subset $C \subseteq V$ such that for every $vw \in E$ it holds that $v \in V \lor w \in V$. In the online setting the vertices are presented one at a time together with all edges to vertices that have been already presented.

Perform an advice preserving reduction from bit guessing to the vertex cover problem. What lower bound on the advice complexity do you get depending on the desirable competitive factor?

Exercise 3
The independent set problem asks for a graph $G = (V, E)$ for a subset $I \subseteq V$ such that for every $v, w \in I$ it holds that $vw \notin E$. In the online setting the vertices are presented one at a time together with all edges to vertices that have been already presented.

Perform an advice preserving reduction from bit guessing to the independent set problem. What lower bound on the advice complexity do you get depending on the desirable competitive factor?

Exercise 4
The cow plans to breakout from the farm together with her cow friends. She plans that all cows should split up, walk around and at a certain time, they all walk to the very long fence and arrive at the same time in front of the very long fence at different positions $c_i$. The cow thinks that it would be the best if every of the $k$ cows leaves through a different gate. There are exactly $k$ gates on the very long fence, but no one knows where. So, every cow marks a gate if she leaves the farm through it and does not use a marked gate.

For this, the cow teaches her walking strategy from Exercise 5 on sheet 2 to the other cows. So, every cow will start to walk one cow meter to the right, then back and two cow meters to the left and so on. The cow knows that cows are good listeners and therefore she knows that every cow will start with the same walking direction and will walk equally fast.

The cow summarizes her thoughts: Each of the $k$ cows should go through a different gate (also $k$ many). The cows start to walk in the same direction with the same speed and the same strategy. She compares the sum of the walked distances by all cows to the sum of the ways that would have minimized the summarized walking distance of all cows and thinks of a worst case. As a cow she does the obvious: She tries not to be the leftmost or the rightmost cow during the breakout.

Explain the possible worst case for this approach. Which number of cows is bad and how should their starting positions be in relation to the gates? Please make a small picture and explain what can happen. You can use the result from Exercise 5 on sheet 2 that one cow walks in the worst case 9 times the optimal distance until it reaches the optimal position. Is the ratio between the optimal solution and the sum of the walked distances still a constant?
Bonus presentation exercises: Write your tutor (fischer@cs.rwth-aachen.de or tarik.viehmann@rwth-aachen.de) a mail and announce that you would like to present a presentation exercise. For every exercise group, only one student is allowed to present an exercise. So, write in your mail which exercise you would like to present and your group number. You are allowed to use the whiteboard and the slides from the lecture.

**Bonus Exercise 5**
Lower and upper Bound for the Simple Knapsack Problem.
Slides: 8:12 to 8:20 (Handout)

**Bonus Exercise 6**
Upper Bound for the Simple $\gamma$-Knapsack problem.
Slides: 8:33 to 8:45 (Handout)

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**Deadline:** The solutions are to be handed in until **July 03, 17:45**, in the lecture or at the drop boxes at the Chair i1.