Exercise 1

Let $C_i$ denote a cycle of length $i$. Prove that for all $k \geq 1$,

$$r_2(C_{2k+1}) = k + 2$$

It may be convenient to start by doing a simple example to get an idea, then prove that the task cannot be done any faster than in $k + 2$ steps in the general case. For the other direction, give an algorithm achieving this bound for all possible $k$ and prove its correctness.

Exercise 2

Exercise 2.1

Let $T_k(m)$ denote a complete tree of depth $m$ in which every non-leaf node has exactly $k$ children (and, of course, all leaves have distance $m$ from the root). Prove or disprove that for $k, m \geq 2$:

$$\text{minb}(T_k(m)) = mk$$

Exercise 2.2

What is $b(T_k(m))$?

Exercise 3

Consider the polynomial-time algorithm from the lecture for special broadcast on trees. Explain a way to distributively implement the first part of the algorithm (i.e., the computation of the broadcasting times for each node) in time $O(\text{depth}(T)(\Delta - 1))$, where $\Delta$ is the maximum degree of any node. Use the communication model from the lecture (telegraph mode, i.e.: Synchronous model where every node can only either send or receive in the same time step, and only to or from one other node). Also, give a short argument for the optimality of the policy in the second part of the algorithm (i.e., sending the information to the subtrees in descending order of their computed broadcasting times).