Exercise 1

The treewidth of a graph $G$ can also be defined via the minimum number of cops needed to succeed in the pursuit-evasion game defined in the lecture. Show that the minimum number of cops needed to always win the game in finite time is $k + 1$ iff $G$'s tree width is $k$. For the "⇒" direction, it is enough to describe informally how a tree decomposition can be derived from a pursuit strategy, and why it fits the definition.

Exercise 2

How must the pursuit-evasion-game from the above exercise be modified to make the number of policemen needed correspond to the path width instead of the tree width of a graph? Explain why.

Exercise 3

Prove that the bandwidth of a graph is lower bounded by its diameter in the following way:

$$bw(G) \geq \lceil \frac{n - 1}{diam(G)} \rceil$$

Exercise 4

What is the bandwidth of a $m \times n$ grid graph? Give a labeling that achieves this bandwidth.

Exercise 5

Consider a $m \times n$ torus graph, i.e. a grid graph with additional edges between the leftmost and rightmost node of each row, and also between the top and bottom node of each column. What could be the bandwidth of such a graph? Again, also give a labeling achieving a good bandwidth.