• Write the name, group number and enrollment number of each group member on every sheet that you hand in.
• To achieve the permission for the exam you must earn 50% of the sum of all points and present one of your solutions at least once.

Exercise 1 (4 points)
Prove that the maximum independent set problem on maximal outerplanar graphs is in $P$.

Exercise 2 (2 points)
Prove or disprove the following statements:
(a) $G = (V, E)$ is planar and Hamiltonian $\Rightarrow \exists E_1, E_2 : E_1 \cup E_2 = E$, and for $i \in \{1, 2\}$, $G_i = (V, E_i)$ is connected and outerplanar.
(b) The opposite direction also holds.
(A graph is called Hamiltonian iff it contains a Hamiltonian cycle, i.e. a simple cycle containing all vertices.)

Exercise 3 (2 points)
Look at the 5-coloring proof for planar graphs and explain why it cannot be applied to the case of a 4-coloring. What could happen in the case where all neighbors of a vertex $v$ with $\deg(v) = 5$ are colored with four different colors. Give an example.

Exercise 4 (4 points)
Let $G = (V, E)$ be a graph, and $f_k$ a function s.t. for each $e \in E$: $f_k(e) \in \{1, \ldots, k\}$. Assume that $G$ can be embedded into the plane such that no two edges $e_1, e_2$ with $f_k(e_1) = f_k(e_2)$ are crossing each other.

For the case $k = 2$ and $n \geq 24$, give an example for which the size of a smallest $(O(\sqrt{n}), \frac{1}{2})$-separator is at least $2[\sqrt{n}] - 1$. Use two different colors to visualize the non crossing property of $f_1$ and $f_2$ in your example.

Bonus presentation exercises: Write your tutor (moritz.goeke@rwth-aachen.de or tarik.viehmann@rwth-aachen.de) a mail and announce that you would like to present a presentation exercise. For every exercise group, only one student is allowed to present an exercise. So, write in your mail which exercise you would like to present and your group number. You are allowed to use the whiteboard and the slides from the lecture.

Bonus Exercise 5 (4 points (bonus))
The 5-color proof for planar graphs.
Slides: 1:39 to 1:40 (Handout)

Bonus Exercise 6 (4 points (bonus))
The hardness proof for 3-coloring on planar graphs with degree $\leq 4$.
Slides: 1:42 to 1:49 (Handout)

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Deadline: The solutions are to be handed in until October 23, 18:00, in the lecture or at the drop boxes at the Chair i1.