Exercise 1 (6 points)

(a) Is there a graph property that is an upper bound for the cyclewidth of arc graphs?
(b) Give a simple and efficient algorithm that computes a 2-approximation of the cyclewidth of an arc graph. Explain your solution.

Hint: The cyclewidth of a clique with \( n \) vertices is \( \lfloor \frac{n}{2} \rfloor \).

Exercise 2 (2 points)

Give an example of a graph \( G \), such that \( \chi(G) = \omega(G) \), but \( G \) is not perfect.

Exercise 3 (2 points)

Prove that interval graphs are \( \chi \)-perfect.

Exercise 4 (4 points)

A \((\alpha, \omega)\)-graph is a graph \( G \) with \( |V(G)| = \alpha(G) \cdot \omega(G) + 1 \) (and \( \alpha(G), \omega(G) \geq 2 \)), such that \( \forall v \in V(G): V(G) \setminus \{v\} \) can both be partitioned into cliques of size \( \omega(G) \) or independent sets of size \( \alpha(G) \).
Show that every minimal imperfect graph is a \((\alpha, \omega)\)-graph.
(This proves the theorem from the lecture 5:21: A graph \( G \) is perfect iff \( \alpha(H) \cdot \omega(H) \geq |V(H)| \) for all induced subgraphs.)

Exercise 5 (4 points)

A Hypercube \( HQ(d) = (V_{HQ(d)}, E_{HQ(d)}) \) of dimension \( d \) is a graph where every vertex is a binary string of length \( d \geq 1 \). Therefore the set of vertices is defined as \( V_{HQ(d)} = \{0, 1\}^d \). An edge connects two vertices if their Hamming distance is equal to one. Thus, the set is defined as \( E_{HQ(d)} = \{\{w0w', w1w'\}|w0w', w1w' \in V_{HQ(d)}\} \).
We are interested in graphs that are constructed like the following:
Given a Hypercube \( HQ(d) \) of dimension \( d \), create for every possible Hypercube that is contained in \( HQ(d) \) a vertex. If two of these Subhypercubes share at least one vertex, they are connected by an edge. Now we create our graphs that we are interested in by taking a node induced subgraph of this graph. Every vertex represents one hypercube that is part of \( HQ(d) \).

For which \( d \) are graphs constructed like this are not \( \chi \)-perfect.

Bonus presentation exercises: Write your tutor (moritz.goeke@rwth-aachen.de or tarik.viehmann@rwth-aachen.de) a mail and announce that you would like to present a presentation exercise. For every exercise group, only one student is allowed to present an exercise. So, write in your mail which exercise you would like to present and your group number. You are allowed to use the whiteboard and the slides from the lecture.

Bonus Exercise 6 (4 points (bonus))
Explanation of the definition of pathwidth and treewidth with some examples.
Slides: 4:24 and 4:31 (Handout)

Bonus Exercise 7 (4 points (bonus))
Treewidth of cactus graphs, near-tree graphs, halin graphs and outer-planar graphs.
Slides: 4:48 to 4:55 (Handout)
<table>
<thead>
<tr>
<th>Exercise</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points:</td>
<td>6</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>Bonus:</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Score:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Deadline:** The solutions are to be handed in until **December 4, 18:00**, in the lecture or at the drop boxes at the Chair i1.