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A graph consists of nodes, which are “connected” by some relation.

Often we have objects, for which some relation exists.

Possible relations:

- Objects have some common property.
- Objects are neighbours.
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- Objects intersect.

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Definition

- A graph $G = (V, E)$ is called intersection-graph of a set $\mathcal{M}$ of objects, iff $G = (V, E)$ is isomorphic to $H = (\mathcal{M}, \{\{a, b\} \mid a \cap b \neq \emptyset\})$.
- $\mathcal{M}$ is called the intersection representation of $G$.

- Possible families of objects are:
  - Intervals on a line.
  - Arc of a circle.
  - Chords of a circle.
  - Circles in the plane.
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- By using different classes of object we get different graph classes.
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Colouring

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- A graph $G = (V, E)$ is $k$-colourable iff:
  - $\exists f : V \mapsto \{1, \ldots, k\} : \forall (a, b) \in E, f(a) \neq f(b)$.
- The function $f$ is called colouring of $G$.

Definition

- $\chi(G)$ is the chromatic number $\chi(G)$ of $G$, iff
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Colouring Problems

Definition

The graph-to-colour problem is the following:
Input: $G$ a graph
Output: Optimal colouring of $G$.

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The colouring problem is the following:
Input: $k \in \mathbb{N}$ and a graph $G$
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**Definition**
- A graph $G = (V, E)$ contains an independent set of size $k$, iff
  \[ \exists S \subseteq V : |S| = k \land \forall a, b \in S, a \neq b : (a, b) \notin E. \]

**Definition**
- $\alpha(G)$ denotes the size of the largest independent set:
  - $G$ contains an independent set of size $\alpha(G)$, but no independent set of size $\alpha(G) + 1$. 
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Definitions

Let $G = (V, E)$ be a graph.

\[ \alpha(G) = \max \{|V'|; \ V' \subset V \land \forall a, b \in V': (a, b) \notin E \} \]

\[ \omega(G) = \max \{|V'|; \ V' \subset V \land \forall a, b \in V': (a, b) \in E \} \]

\[ \chi(G) = \min \{k; \ \exists V_1, V_2, \ldots, V_k : \bigcup_{i=1}^{k} V_i = V \land \forall i: 1 \leq i \leq k : \forall a, b \in V_i : (a, b) \notin E \} \]

\[ \bar{\chi}(G) = \min \{k; \ \exists V_1, V_2, \ldots, V_k : \bigcup_{i=1}^{k} V_i = V \land \forall i: 1 \leq i \leq k : \forall a, b \in V_i : (a, b) \in E \} \]

More notations:

\[ \omega(G) = \overline{\alpha}(G), \]
\[ \alpha(G) = \overline{\omega}(G) = \beta_0(G), \]
\[ \kappa(G) = \overline{\chi}(G) \]
First simple Example

- **Time of activity of a register (construction of a compiler)**
- **Program segments:** \( \cdots \text{Read}(A) \cdots \text{Write}(B) \cdots \)
- **Living time of a variable** A: Maximal interval
  - Starting with a \( \text{Write}(A) \).
  - Ending by the last \( \text{Read}(A) \).
  - Such that no further \( \text{Write}(A) \) is between this two points.

- **Problem:** how many registers are needed?
- **D.h.** assign for each living time of a variable a register.
- **Example:**  
  \((0, 10), (3, 7), (9, 20), (25, 50), (12, 34), (6, 16), (17, 26), (11, 46), (23, 26), (30, 46), (19, 27)\)
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- it is the intersection graph of a set of intervals on a line.
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Model and Colouring (Idea)

Idea: look for independent sets.
Model and Colouring (Idea)

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0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50

- a
- b
- c
- d
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Model and Colouring (Invariant)

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Model and Colouring (Invariant)

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![Graph representation of the invariant](image-url)
Model and Colouring (Invariant)

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The diagram shows intervals on a number line from 0 to 50, with certain intervals highlighted in different colors, indicating the invariant.
Model and Colouring (Invariant)

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+------------------+
|                 |
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---

Model and Colouring (Invariant)
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Determine the invariant:
Model and Colouring (Invariant)

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![Diagram showing colouring of intervals on a timeline]
Theorem

The graph-to-colour problem is for interval-graphs in time $O(n \log(n))$ solvable.

1. Sort the intervals by their left endpoints.
2. Check all endpoints $e$ from the left to the right.
3. If $e$ is the starting point of an interval, colour it with the smallest free colour.
4. If $e$ is the ending point of an interval $I$ is, free the colour of $I$.

Invariant

If a node $v$ is coloured with colour $k$, then $v$ is part of a $k$-clique.
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Example of independent set problem on interval-graphs

1. Sort the intervals by their starting points.
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**Theorem**

*Finding a maximal independent set is solvable in time $O(n \log(n))$ on interval-graphs.*

1. Sweep through the start- and endpoints of intervals from left to right.
2. Store for each endpoint $e$ the size of a maximal independent set of intervals, which is placed to the left of $e$.
3. While sweeping from left to right do:
   1. If $e$ is a starting point of interval $(e, f)$ and there is no endpoint to the left of $e$, then let $S(f) = 1$.
   2. If $e$ is a starting point of interval $(e, f)$, then compute:
      - largest endpoint $e'$ to the left of $e$ and let $S(f) = S(e') + 1$.
   3. If $e$ is an endpoint of interval $(a, e)$, then compute:
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Maximal Clique on Interval-graphs

**Theorem**

*Finding a maximal clique is solvable in time* $O(n \log(n))$ *on interval-graphs.*

**Remark**

Very many problems are efficient solvable on interval-graphs.
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Definition (Permutations-Graph)

- A graph $G = (V, E)$ is called permutation-graph,
- iff it is definable by a permutation $\pi : \{1..n\} \rightarrow \{1..n\}$ in the following way:
- $G = (\{1..n\}, \{(i,j); (i - j)(\pi(i) - \pi(j)) < 0\})$.

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A permutation-graph is the intersection graph of a set of lines, which are drawn between to parallel lines.
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- $G = (\{1..n\}, \{(i, j); (i - j)(\pi(i) - \pi(j)) < 0\})$.

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A permutation-graph is the intersection graph of a set of lines, which are drawn between two parallel lines.
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Example and Colouring

The invariant is the same as the one on interval-graphs.
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Example and Colouring

![Graph with nodes and edges colored with different colors]
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Colouring Problem on Permutation-Graphs

**Theorem**

*The graph-to-colour problem is solvable in time $O(n \log(n))$ on permutation-graphs.*

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Definition (Arc-Graph)

- A graph $G = (V, E)$ is called arc-graph,
- iff he is the intersection graph of a set of arcs on a circle.
- A arc-graph is called proper, iff no arc in contained in an other arc.

Remark

An interval-graph is an arc-graph.

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Question, what is the reason that the above problems are efficient solvable on interval-graphs?

Consider the "flow of information", i.e.:

Which information is used (stored) when the algorithms move from left to right.

One could think, all $k!$ colourings should be considered (stored).

But, the colourings are exchangeable.

Thus only the optimal colouring at each position is stored.

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What is the situation on arc-graphs?
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**Question:**

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Colouring on Arc-Graphs (Idea)

- Consider the flow of information.
- What information has to be considered when moving around the circle?
- The colouring are not exchangeable because the end the colours have to match.
- Thus we may have to consider $k!$ colourings.
- If $k$ is constant, then the problem is in $\mathcal{P}$
- IF $k$ is not constant, then the problem could be in $\mathcal{NPC}$.
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Theorem

*The k-colouring problem on arc-graphs is solvable in polynomial time.*

Idea: Consider all $k!$ colourings.

1. W.l.o.g.: The graph contains no $k + 1$ clique.
2. Otherwise we search analog as on interval-graphs for the largest clique.
3. Colour an some maximal $k'$-Clique.
4. Colour the arcs in a clockwise order.
5. At most $k!$ colourings are considered (stored) during this process.
6. Check at the end if some colouring do not contradict with the first one.
7. Running time: $O(k!^2 \cdot n \log n) = O(n \log n)$
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The colouring problem on arc-graphs NP-complete.

Idea: Reduction to the word problem for symmetric groups.

The word problem for symmetric groups is the following:

**Input:** \( \pi \in S_k \) (Word and symmetric group) and \( S_1, S_2, \cdots, S_n \) subgroups

**Output:** Holds: \( \pi \in S_1 \circ S_2 \circ \cdots \circ S_n \)
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Colouring Problem on Arc-Graphs

S1 = {2, 4}
S2 = {4, 6}
S3 = {1, 3}
S4 = {1, 6}

π(1) = 3
π(2) = 1
π(3) = 2
π(4) = 5
π(5) = 4
π(6) = 6
Colouring Problem on Arc-Graphs

\[
S_1 = \{2, 4\}
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\[ \pi(5) = 4 \]

\[ \pi(6) = 6 \]

[Diagram showing the colouring process on an arc-graph with sets \( S_1 = \{2, 4\} \), \( S_2 = \{4, 6\} \), \( S_3 = \{1, 3\} \), and \( S_4 = \{1, 6\} \).]
Colouring Problem on Arc-Graphs

\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
\[ S_3 = \{1, 3\} \]
\[ S_4 = \{1, 6\} \]

\[ \pi(1) = 3 \]
\[ \pi(2) = 1 \]
\[ \pi(3) = 2 \]
\[ \pi(4) = 5 \]
\[ \pi(5) = 4 \]
\[ \pi(6) = 6 \]
Colouring Problem on Arc-Graphs

\[ \pi(2) = 3 \]
\[ \pi(1) = 1 \]

\[ \pi(1) = 3 \]
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\[ \pi(3) = 2 \]
\[ \pi(4) = 5 \]
\[ \pi(5) = 4 \]
\[ \pi(6) = 6 \]

\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
\[ S_3 = \{1, 3\} \]
\[ S_4 = \{1, 6\} \]
Colouring Problem on Arc-Graphs

\[ \pi(1) = 3 \]
\[ \pi(2) = 1 \]

\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
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Colouring Problem on Arc-Graphs

\[ \pi(3) = 2 \]
\[ \pi(2) = 1 \]
\[ \pi(1) = 3 \]

\( S_1 = \{2, 4\} \)
\( S_2 = \{4, 6\} \)
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Colouring Problem on Arc-Graphs

\[ \pi(1) = 3 \]
\[ \pi(2) = 1 \]
\[ \pi(3) = 2 \]

\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
\[ S_3 = \{1, 3\} \]
\[ S_4 = \{1, 6\} \]
Colouring Problem on Arc-Graphs

\[ \pi(4) = 5 \]
\[ \pi(3) = 2 \]
\[ \pi(2) = 1 \]
\[ \pi(1) = 3 \]
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\[ \pi(1) = 3 \]
\[ \pi(2) = 1 \]
\[ \pi(3) = 2 \]
\[ \pi(4) = 5 \]

\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
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Colouring Problem on Arc-Graphs

\[ \pi(5) = 4 \]
\[ \pi(4) = 5 \]
\[ \pi(3) = 2 \]
\[ \pi(2) = 1 \]
\[ \pi(1) = 3 \]

\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
\[ S_3 = \{1, 3\} \]
\[ S_4 = \{1, 6\} \]
Colouring Problem on Arc-Graphs

\[\pi(5) = 4, \pi(4) = 5, \pi(3) = 2, \pi(2) = 1, \pi(1) = 3\]

\[S_1 = \{2, 4\}, S_2 = \{4, 6\}, S_3 = \{1, 3\}, S_4 = \{1, 6\}\]
Colouring Problem on Arc-Graphs

\[ \pi(1) = 3 \]
\[ \pi(2) = 1 \]
\[ \pi(3) = 2 \]
\[ \pi(4) = 5 \]
\[ \pi(5) = 4 \]
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\[ S_1 = \{2, 4\} \]
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\[ \pi(6) = 6 \]
\[ \pi(5) = 4 \]
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\[ \pi(3) = 2 \]
\[ \pi(2) = 1 \]
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\[ \pi(5) = 5 \]
\[ \pi(4) = 2 \]
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Definition (Circle-Graphs)

- A graph $G = (V, E)$ is called circle-graph,
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- A graph $G = (V, E)$ is called overlap-graph,
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- Let $I$ be a set of intervals.
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**Lemma**

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- What is the flow of information?
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Coloring Problems (Overview)

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  - For a given 3-SAT formula $\mathcal{F}$ we construct a circle-graph $G$.
  - It has to hold: $\mathcal{F}$ satisfiable $\iff$ $G$ 4-colourable.
  - Problem: Coding of logical values by the colouring of cords.
  - Idea: Each pair of chord $(a, b)$ codes a logical value of $v$.
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It has to hold: \( \mathcal{F} \) satisfiable \( \iff \) \( G \) 4-colourable.

Problem: Coding of logical values by the colouring of cords.

Idea: Each pair of chord \((a, b)\) codes a logical value of \( v \).

Holding: \( v \iff f(a) = f(b) \) for a colouring \( f \).

Construct some kind of “circuit”.
4-Colouring Problem on Circle-Graphs

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- Idea: Each pair of chord $(a, b)$ codes a logical value of $\nu$.
- Holding: $\nu \iff f(a) = f(b)$ for a colouring $f$.
- Construct some kind of “circuit”.
Component Negation I \((x = \neg y)\)
Component Negation 1 ($x = \neg y$)
Component Negation $1 (x = \neg y)$
Component Negation I \((x = \neg y)\)
Component Negation I \((x = \neg y)\)
Component Negation 1 \((x = \neg y)\)
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Component Negation I ($x = \neg y$)
Component Negation 1 \((x = \neg y)\)
Component Negation I \((x = \neg y)\)
Component Negation I \( (x = \neg y) \)
Overview
Overview
The Negation

Negation II: $x = \neg y$

Combination of Colours

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The Negation

Negation II: $x = \neg y$

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Negation II: $x = \neg y$

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Negation II: \( x = \neg y \)

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The Negation

Negation II: \( x = \neg y \)

![Diagram of negation]

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Combination of Colours
The Negation

Negation II: $x = \neg y$

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Some Simple Components

Negation II:

\[ x = \neg y \]
Some Simple Components

Negation II:

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Some Simple Components

Negation II:
\[ x = \neg y \]

Equality:
\[ x = y \]
Some Simple Components

Negation II:
\[ x = \neg y \]

Equality:
\[ x = y \]
### Some Simple Components

**Negation II:**

\[ x = \neg y \]

**Equality:**

\[ x = y \]

**Static XOR:**

\[ x = y \oplus e \]
Some Simple Components

Negation II:
\[ x = \neg y \]

Equality:
\[ x = y \]

Static XOR:
\[ x = y \oplus e \]
Equality: \((x = y = z)\)

- \(\neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z\)
- \(\neg y \Rightarrow b_1 \Rightarrow \neg x\)
- \(y \Rightarrow \neg a_1 \Rightarrow \neg a_2 \Rightarrow z\)
- \(y \Rightarrow \neg a_2 \Rightarrow b_2 \Rightarrow \neg b_1 \Rightarrow x\)
- A colouring is possible in all cases.
Equality: \((x = y = z)\)

\[
\begin{align*}
\neg y & \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z \\
\neg y & \Rightarrow b_1 \Rightarrow \neg x \\
y & \Rightarrow \neg a_1 \Rightarrow \neg a_2 \Rightarrow z \\
y & \Rightarrow \neg a_2 \Rightarrow b_2 \Rightarrow \neg b_1 \Rightarrow x \\
\text{A colouring is possible in all cases.}
\end{align*}
\]
Equality: \((x = y = z)\)

- \(\neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z\)
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Equality: \((x = y = z)\)

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Equality: \( (x = y = z) \)

- \( \neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z \)
- \( \neg y \Rightarrow b_1 \Rightarrow \neg x \)
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- \(y \Rightarrow \neg a_2 \Rightarrow b_2 \Rightarrow \neg b_1 \Rightarrow x\)
- A colouring is possible in all cases.
Equality: \( (x = x' \land y = y') \)
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Equality: \((x = x' \land y = y')\)

\[
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\end{array}
\]
Equality: \((x = x' \land y = y')\)
Equality: \((x = x' \land y = y')\)
Equality ($x = y = z$)

$x = y = z$
Equality \((x = y = z)\)

\[ x = y = z \]

\[ x = x' \] and \[ y = y' \]
Equality \((x = y = z)\)

\[ x = y = z \]

\[ x = x' \text{ and } y = y' \]
Equality \((x = y = z)\)
More Simple Components

Weak Or:
\[ \neg x \land \neg z \Rightarrow \neg y \]
More Simple Components

Weak Or:
\[ \neg x \land \neg z \Rightarrow \neg y \]
More Simple Components

Weak Or:
\[ \neg x \land \neg z \Rightarrow \neg y \]

Weak Negation:
\[ \neg x \Rightarrow y \text{ and } \neg y \Rightarrow x \]
More Simple Components

Weak Or:
\[ \neg x \land \neg z \Rightarrow \neg y \]

Weak Negation:
\[ \neg x \Rightarrow y \text{ and } \neg y \Rightarrow x \]
More Simple Components

Weak Or:
\( \neg x \land \neg z \Rightarrow \neg y \)

Weak Negation:
\( \neg x \Rightarrow y \) and \( \neg y \Rightarrow x \)

True:
\( x = true \)
More Simple Components

Weak Or:
\[ \neg x \land \neg z \Rightarrow \neg y \]

Weak Negation:
\[ \neg x \Rightarrow y \text{ and } \neg y \Rightarrow x \]

True:
\[ x = true \]
More Simple Components

Weak Or:
\[ \neg x \land \neg z \Rightarrow \neg y \]

Weak Negation:
\[ \neg x \Rightarrow y \text{ and } \neg y \Rightarrow x \]

True:
\[ x = true \]
Or $(x \lor y = z)$

- $\neg x \land \neg y \Rightarrow \neg z_3 \land \neg y_1 \Rightarrow \neg z$.
- $x \Rightarrow \neg x' \Rightarrow z_1 \Rightarrow z$.
- $y \Rightarrow \neg y' \Rightarrow z_4 \Rightarrow z$.
- A colouring is possible in all cases.
Or \((x \lor y = z)\)

\[
\begin{align*}
\neg x \land \neg y & \Rightarrow \neg x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z \\
x & \Rightarrow \neg x' \Rightarrow z_1 \Rightarrow z \\
y & \Rightarrow \neg y' \Rightarrow z_4 \Rightarrow z
\end{align*}
\]

A colouring is possible in all cases.
Or \((x \lor y = z)\)

- \(\neg x \land \neg y \Rightarrow x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z\)
- \(x \Rightarrow \neg x' \Rightarrow z_1 \Rightarrow z\)
- \(y \Rightarrow \neg y' \Rightarrow z_4 \Rightarrow z\)
- A colouring is possible in all cases.
Or \((x \lor y = z)\)

- \(-x \land y \Rightarrow \neg x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z\)
- \(x \Rightarrow \neg x' \Rightarrow z_1 \Rightarrow z\)
- \(y \Rightarrow \neg y' \Rightarrow z_4 \Rightarrow z\)
- A colouring is possible in all cases.
Or \((x \lor y = z)\)

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Static Simple Clause

\[ x_2 = x_2' \quad \text{and} \quad (x_1 \oplus e_1) \lor (x_2 \oplus e_2) \lor (x_3 \oplus e_3) = true \]
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\[ x_2 = x'_2 \text{ and } (x_1 \oplus e_1) \lor (x_2 \oplus e_2) \lor (x_3 \oplus e_3) = true \]
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The $k$-colouring problem on circle-graphs is NP-complete for $k \geq 4$.

Theorem

The $(2 \cdot k - 1)$-colouring problem on circle-graphs with clique size $k$ is NP-complete for $k \geq 3$.

Theorem

A circle-graph with clique size $k$ is always $(3 \cdot k)$-colourable.
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Theorem

On an interval graph $G$ we may in time $O(n \log(n))$ compute $\chi(G)$, $\alpha(G)$ and $\omega(G)$.

Theorem

On a permutation graph $G$ we may in time $O(n \log(n))$ compute $\chi(G)$, $\alpha(G)$ and $\omega(G)$.

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g-Segment-graphs

Definition (g-Segment-graphs)

- A graph $G = (V, E)$ is called $g$-Segment-graph, iff
- it is the intersection-graph of a set of chords within a regular $g$-polygon.

Lemma

We have:

1. A permutation-graph is a circle-graph.
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• How is the colouring problem solvable on permutation graphs?
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Legend

- : Not of relevance
- : implicitly used basics
- : idea of proof or algorithm
- : structure of proof or algorithm
- : Full knowledge