Algorithmic Graph Theory (SS2016)
Chapter 3
Simple Intersection-Graphs

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- Often we have objects, for which some relation exists.
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  - Objects intersect.
- We define intersection-graphs using the later relation.
Definition

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Possible families of objects are:
- Intervals on a line.
- Arcs of a circle.
- Chords of a circle.
- Circles in the plane.
- Parallelograms between two lines.
- And lots more.

By using different classes of objects we get different graph classes.
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By using different classes of object we get different graph classes.
Colouring

**Definition**

- A graph $G = (V, E)$ is $k$-colourable iff:
  - $\exists f : V \mapsto \{1, \ldots, k\} : \forall (a, b) \in E, f(a) \neq f(b)$.
- The function $f$ is called colouring of $G$.

**Definition**

- $\chi(G)$ is the chromatic number $\chi(G)$ of $G$, iff
  - $G$ is $\chi(G)$-colourable, but is not $(\chi(G) - 1)$-colourable.
**Definition**

The graph-to-colour problem is the following:

**Input:** $G$ a graph

**Output:** Optimal colouring of $G$.

**Definition**

The colouring problem is the following:

**Input:** $k \in \mathbb{N}$ and a graph $G$

**Output:** Is $G$ $k$-colourable?

**Definition**

The $k$-colouring problem is the following:

**Input:** $G$ a Graph

**Output:** Is $G$ $k$-colourable?
Definition

- A graph $G = (V, E)$ contains an independent set of size $k$, iff
- $\exists S \subseteq V : |S| = k \land \forall a, b \in S, a \neq b : (a, b) \notin E.$

Definition

- $\alpha(G)$ denotes the size of the largest independent set:
- $G$ contains an independet set of size $\alpha(G)$, but no independet set of size $\alpha(G) + 1$. 
Definitions

Let $G = (V, E)$ be a graph.

$$\alpha(G) = \max\{ |V'| ; \ V' \subseteq V \land \forall a, b \in V' : (a, b) \notin E \}$$

$$\omega(G) = \max\{ |V'| ; \ V' \subseteq V \land \forall a, b \in V' : (a, b) \in E \}$$

$$\chi(G) = \min\{ k ; \ \exists V_1, V_2, \ldots, V_k : \bigcup_{i=1}^{k} V_i = V \land$$
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$$\overline{\chi}(G) = \min\{ k ; \ \exists V_1, V_2, \ldots, V_k : \bigcup_{i=1}^{k} V_i = V \land$$
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More notations:

$\omega(G) = \overline{\alpha}(G)$

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$\kappa(G) = \overline{\chi}(G)$
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Theorem

The graph-to-colour problem is for interval-graphs in time $O(n \log(n))$ solvable.
Colouring of Interval-graphs (Algorithm)

Theorem

*The graph-to-colour problem is for interval-graphs in time $O(n \log n)$ solvable.*

1. Sort the intervals by their left endpoints.
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The graph-to-colour problem is for interval-graphs in time $O(n \log(n))$ solvable.

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2. Check all endpoints $e$ from the left to the right.
**Theorem**

The graph-to-colour problem is for interval-graphs in time $O(n \log(n))$ solvable.

1. Sort the intervals by their left endpoints.
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Invariant

If a node $v$ is coloured with colour $k$, then $v$ is part of a $k$-clique.
Example of independent set problem on interval graphs

Sort the intervals by their starting points.

Go through all starting points from left to right.

Store for each interval the size of a maximal independent set of intervals, which contain the interval as the rightmost interval.
Example of independent set problem on interval-graphs

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Theorem

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Independent Set Problem for Interval-graphs

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1. Sweep through the start- and endpoints of intervals from left to right.
Independent Set Problem for Interval-graphs

**Theorem**

*Finding a maximal independent set is solvable in time $O(n \log(n))$ on interval-graphs.*

1. **Sweep through the start- and endpoints of intervals from left to right.**
2. **Store for each endpoint $e$ the size of a maximal independent set of intervals, which is placed to the left of $e$.**
Independent Set Problem for Interval-graphs

**Theorem**

_Finding a maximal independent set is solvable in time \(O(n \log(n))\) on interval-graphs._

1. Sweep through the start- and endpoints of intervals from left to right.
2. Store for each endpoint \(e\) the size of a maximal independent set of intervals, which is placed to the left of \(e\).
3. While sweeping from left to right do:
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   1. If $e$ is a starting point of interval $(e, f)$ and there is no endpoint to the left of $e$, then let $S(f) = 1$. 

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   2. If $e$ is a starting point of interval $(e, f)t$, then compute:
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   2. If $e$ is a starting point of interval $(e, f)t$, then compute: largest endpoint $e'$ to the left of $e$ and let $S(f) = S(e') + 1$.
   3. If $e$ is an endpoint of interval $(a, e)$, then compute: largest endpoint $e'$ to the left of $e$ and to the right of $a$. If that exists, then let $S(e) = \max(S(e'), S(e))$. 
Maximal Clique on Interval-graphs

**Theorem**

*Finding a maximal clique is solvable in time $O(n \log(n))$ on interval-graphs.*
Maximal Clique on Interval-graphs

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**Remark**

Very many problems are efficient solvable on interval-graphs.
Definition (Permutations-Graph)

A graph \( G = (V, E) \) is called permutation-graph,
Permutation-Graphs

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- A graph $G = (V, E)$ is called permutation-graph,
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Theorem
A permutation-graph is the intersection graph of a set of lines, which are drawn between two parallel lines.
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Example and Colouring

\[ \pi(i) \quad \pi(h) \quad \pi(e) \quad \pi(c) \quad \pi(f) \quad \pi(a) \quad \pi(b) \quad \pi(d) \quad \pi(k) \quad \pi(g) \quad \pi(j) \]

\[ a \quad b \quad c \quad d \quad e \quad f \quad g \quad h \quad i \quad j \quad k \]

\[ b \quad c \quad e \quad h \]

The invariant is the same as the one on interval-graphs.
Example and Colouring

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\[ \sum = 0 \]

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a  b  c  d  e  f  g  h  i  j  k

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The invariant is the same as the one on intervall-graphs.
Colouring Problem on Permutation-Graphs

**Theorem**

The graph-to-colour problem is solvable in time $O(n \log(n))$ on permutation-graphs.

Idea: Analog algorithms as on interval-graphs.
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Arc-Graphs

Definition (Arc-Graph)

A graph $G = (V, E)$ is called arc-graph,
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- A graph $G = (V, E)$ is called arc-graph,
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A arc-graph is called proper, iff no arc in contained in an other arc.
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An interval-graph is an arc-graph.
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Remark

An interval-graph is an arc-graph.

Question:

Are the algorithms for interval-graphs adaptable to arc-graphs.
Reasoning for the above Results

- Question, what is the reason that the above problems are efficient solvable on interval-graphs?
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Question:

What is the situation on arc-graphs?
Consider the flow of information.
Colouring on Arc-Graphs (Idea)

- Consider the flow of information.
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If $k$ is constant, then the problem is in $\mathcal{P}$. 
Colouring on Arc-Graphs (Idea)

- Consider the flow of information.
- What information has to be considered when moving around the circle?
- The colouring are not exchangeable because the end the colours have to match.
- Thus we may have to consider $k!$ colourings.
- If $k$ is constant, then the problem is in $\mathcal{P}$
- If $k$ is not constant, then the problem could be in $\mathcal{NPC}$. 
Theorem

The $k$-colouring problem on arc-graphs is solvable in polynomial time.

Idea: Consider all $k!$ colourings.

1. W.l.o.g.: The graph contains no $k + 1$ clique.
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1. W.l.o.g.: The graph contains no \( k + 1 \) clique.
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3. Colour an some maximal \( k' \)-Clique.
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4. Colour the arcs in a clockwise order.
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3. Colour an some maximal $k'$-Clique.
4. Colour the arcs in a clockwise order.
5. At most $k!$ colourings are considered (stored) during this process.
6. Check at the end if some colouring do not contradict with the first one.
7. Running time: $O(k!^2 \cdot n \log n) = O(n \log n)$
The colouring problem on arc-graphs NP-complete.
Theorem

The colouring problem on arc-graphs NP-complete.

Idea: Reduction to the word problem for symmetric groups.

Definition

The word problem for symmetric groups is the following:
Input: $\pi \in S_k$ (Word and symmetric group) and $S_1, S_2, \ldots, S_n$ subgroups
Output: Holds: $\pi \in S_1 \circ S_2 \circ \cdots \circ S_n$
Theorem

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\[ S_1 = \{2, 4\} \]
Colouring Problem on Arc-Graphs

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Colouring Problem on Arc-Graphs

$S_1 = \{2, 4\}$

$\pi(1) = 3$
$\pi(2) = 1$
$\pi(3) = 2$
$\pi(4) = 5$
$\pi(5) = 4$
$\pi(6) = 6$
Colouring Problem on Arc-Graphs

\[ S_1 = \{2, 4\} \]
Colouring Problem on Arc-Graphs

...
Colouring Problem on Arc-Graphs

\[ S_1 = \{2, 4\} \]

\[ S_2 = \{4, 6\} \]
Colouring Problem on Arc-Graphs

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\[ S_2 = \{4, 6\} \]
\[ S_3 = \{1, 3\} \]
\[ S_4 = \{1, 6\} \]
Colouring Problem on Arc-Graphs

\[
\begin{align*}
\pi(1) &= 3 \\
\pi(2) &= 1 \\
\pi(3) &= 2 \\
\pi(4) &= 5 \\
\pi(5) &= 4 \\
\pi(6) &= 6
\end{align*}
\]
Colouring Problem on Arc-Graphs

Let $S_1 = \{2, 4\}$, $S_2 = \{4, 6\}$, $S_3 = \{1, 3\}$, and $S_4 = \{1, 6\}$. The colouring problem can be illustrated as follows:

- $S_1$: Coloured in blue and green.
- $S_2$: Coloured in blue, green, red, and yellow.
- $S_3$: Coloured in blue, green, red, yellow, and magenta.
- $S_4$: Coloured in blue, green, red, yellow, magenta, and cyan.

The arrows indicate the direction of the arcs.
Colouring Problem on Arc-Graphs

\[
\pi(1) = 3 \\
\pi(2) = 1 \\
\pi(3) = 2 \\
\pi(4) = 5 \\
\pi(5) = 4 \\
\pi(6) = 6
\]

\[
S_1 = \{2, 4\} \\
S_2 = \{4, 6\} \\
S_3 = \{1, 3\} \\
S_4 = \{1, 6\}
\]
Colouring Problem on Arc-Graphs

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Colouring Problem on Arc-Graphs

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\[ S_2 = \{4, 6\} \]
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Colouring Problem on Arc-Graphs

\[ \pi(1) = 3 \]

\[ \pi(2) = 1 \]

\[ \pi(3) = 2 \]

\[ \pi(4) = 5 \]

\[ \pi(5) = 4 \]

\[ \pi(6) = 6 \]
Colouring Problem on Arc-Graphs

\[ \pi(1) = 3 \]

\[ \pi(2) = 1 \]

\[ \pi(3) = 2 \]

\[ \pi(4) = 5 \]

\[ \pi(5) = 4 \]

\[ \pi(6) = 6 \]
Colouring Problem on Arc-Graphs

\[ \pi(2) = 1 \]
\[ \pi(1) = 3 \]

\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
\[ S_3 = \{1, 3\} \]
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Colouring Problem on Arc-Graphs

\[ \pi(1) = 3 \]
\[ \pi(2) = 1 \]

\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
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Colouring Problem on Arc-Graphs

\[ \pi(3) = 2 \]
\[ \pi(2) = 1 \]
\[ \pi(1) = 3 \]

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Colouring Problem on Arc-Graphs

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\begin{align*}
\pi(1) &= 3 \\
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Coloursing Problem on Arc-Graphs

\[ \pi(4) = 5 \]
\[ \pi(3) = 2 \]
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\[ S_1 = \{2, 4\} \]
\[ S_2 = \{4, 6\} \]
\[ S_3 = \{1, 3\} \]
\[ S_4 = \{1, 6\} \]
Colouring Problem on Arc-Graphs

\[ \pi(4) = 5, \quad \pi(3) = 2, \quad \pi(2) = 1, \quad \pi(1) = 3 \]

\[ S_1 = \{2, 4\}, \quad S_2 = \{4, 6\}, \quad S_3 = \{1, 3\}, \quad S_4 = \{1, 6\} \]
Colouring Problem on Arc-Graphs

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Colouring Problem on Arc-Graphs

\[
\begin{align*}
\pi(5) &= 4 \\
\pi(4) &= 5 \\
\pi(3) &= 2 \\
\pi(2) &= 1 \\
\pi(1) &= 3
\end{align*}
\]

\[\pi(1) = 3 \quad \pi(2) = 1 \quad \pi(3) = 2 \quad \pi(4) = 5 \quad \pi(5) = 4\]
Colouring Problem on Arc-Graphs

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\begin{align*}
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\end{align*}
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\[ \pi(6) = 6 \]
\[ \pi(5) = 4 \]
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Definition (Circle-Graphs)

A graph \( G = (V, E) \) is called circle-graph,
**Definition (Circle-Graphs)**

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- A graph $G = (V, E)$ is called overlap-graph,
- iff it is definable by the overlapping of a set of intervals on a line.
- Let $I$ be a set of intervals.
- Then the corresponding overlap-graph is:
  
  $G = (I, \{(a, b) \mid a, b \in I \land a \setminus b \neq \emptyset \land b \setminus a \neq \emptyset \land a \cap b \neq \emptyset\})$
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Statements on Circle-Graphs

Lemma

1. *An interval-graph is an arc-graph.*
Statements on Circle-Graphs

Lemma

1. An interval-graph is an arc-graph.
2. A proper arc-graph is a circle-graph.
Statements on Circle-Graphs

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Just show: a graph \( G \) is a circle-graph, iff \( G \) is a overlap-graph.

- Chord \( A \) from \( r \cdot e^{i \cdot a} \) to \( r \cdot e^{i \cdot a'} \) becomes interval \( A' = (a, a') \) \((0 \leq a < a' < 2 \cdot \pi)\).
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Colouring of Circle-Graphs (Idea)

- What is the flow of information?
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- But: information about the colouring of pairs of chords could be an idea.
- Thus, the 4-colouring problem on circle-graphs could be NP-complete.
Colouring of Circle-Graphs (Idea)

- What is the flow of information?
- Crossing chords “limit” the flow of information.
- But: information about the colouring of pairs of chords could be an idea.
- Thus, the 4-colouring problem on circle-graphs could be NP-complete.
- And the 3-colouring on circle-graphs could still be in \( P \).
Theorem

The 4-colouring problem on circle-graphs is NP-complete.
The 4-colouring problem on circle-graphs is NP-complete.

The 3-colouring problem on circle-graphs is solvable in time $O(n \log(n))$. 
4-Colouring Problem on Circle-Graphs

- Reduction from the 3-SAT Problem.
4-Colouring Problem on Circle-Graphs

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- For a given 3-SAT formula $\mathcal{F}$ we construct a circle-graph $G$. 
4-Colouring Problem on Circle-Graphs

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- It has to hold: $\mathcal{F}$ satisfiable $\iff$ $G$ 4-colourable.
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- It has to hold: $F$ satisfiable $\iff$ $G$ 4-colourable.
- Problem: Coding of logical values by the colouring of cords.
- Idea: Each pair of chord $(a, b)$ codes a logical value of $v$.
- Holding: $v \iff f(a) = f(b)$ for a colouring $f$.
- Construct some kind of “circuit”.
Component Negation I \((x = \neg y)\)
Component Negation I \((x = \neg y)\)
Component Negation I \( (x = \neg y) \)
Component Negation I \((x = \neg y)\)
Component Negation I \((x = \neg y)\)
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Overview
Overview
The Negation

Negation II: $x = \neg y$

Combination of Colours

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  & a & b \\
x & \neg & \neg \\
y & \neg & \\
\end{array}
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The Negation

Negation II: \( x = \neg y \)

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Some Simple Components

Negation II:
\[ x = \neg y \]
Some Simple Components

Negation II:
\[ x = \neg y \]
Some Simple Components

Negation II: 
\[ x = \neg y \]

Equality: 
\[ x = y \]
Some Simple Components

Negation II:
\[ x = \neg y \]

Equality:
\[ x = y \]
Some Simple Components

Negation II: $x = \neg y$

Equality: $x = y$

Static XOR: $x = y \oplus e$
Some Simple Components

Negation II:
\[ x = \neg y \]

Equality:
\[ x = y \]

Static XOR:
\[ x = y \oplus e \]
Equality: \((x = y = z)\)
Equality: \((x = y = z)\)
Equality: \((x = y = z)\)

\[
\neg y \Rightarrow a_1 \Rightarrow a_2
\]
Equality: \((x = y = z)\)

\[ \neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z \]
Equality: \((x = y = z)\)

- \(\neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z\)
- \(\neg y\)
Equality: \((x = y = z)\)

- \(\neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z\)
- \(\neg y \Rightarrow b_1\)
Equality: \((x = y = z)\)

- \(\neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z\)
- \(\neg y \Rightarrow b_1 \Rightarrow \neg x\)
Equality: \((x = y = z)\)

- \(\neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z\)
- \(\neg y \Rightarrow b_1 \Rightarrow \neg x\)
- \(y\)
Equality: \( x = y = z \)

- \( \neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z \)
- \( \neg y \Rightarrow b_1 \Rightarrow \neg x \)
- \( y \Rightarrow \neg a_1 \)
Equality: \((x = y = z)\)

- \(\neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z\)
- \(\neg y \Rightarrow b_1 \Rightarrow \neg x\)
- \(y \Rightarrow \neg a_1 \Rightarrow \neg a_2\)
Equality: \((x = y = z)\)

\[

\begin{align*}
\neg y & \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z \\
\neg y & \Rightarrow b_1 \Rightarrow \neg x \\
y & \Rightarrow \neg a_1 \Rightarrow \neg a_2 \Rightarrow z
\end{align*}
\]
Equality: \((x = y = z)\)

- \(\neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z\)
- \(\neg y \Rightarrow b_1 \Rightarrow \neg x\)
- \(y \Rightarrow \neg a_1 \Rightarrow \neg a_2 \Rightarrow z\)
- \(y\)
Equality: \((x = y = z)\)

- \(\neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z\)
- \(\neg y \Rightarrow b_1 \Rightarrow \neg x\)
- \(y \Rightarrow \neg a_1 \Rightarrow \neg a_2 \Rightarrow z\)
- \(y \Rightarrow \neg a_2\)
Equality: \((x = y = z)\)

- \(\neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z\)
- \(\neg y \Rightarrow b_1 \Rightarrow \neg x\)
- \(y \Rightarrow \neg a_1 \Rightarrow \neg a_2 \Rightarrow z\)
- \(y \Rightarrow \neg a_2 \Rightarrow b_2\)
Equality: \((x = y = z)\)

- \(\neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z\)
- \(\neg y \Rightarrow b_1 \Rightarrow \neg x\)
- \(y \Rightarrow \neg a_1 \Rightarrow \neg a_2 \Rightarrow z\)
- \(y \Rightarrow \neg a_2 \Rightarrow b_2 \Rightarrow \neg b_1\)

A colouring is possible in all cases.
Equality: \((x = y = z)\)

- \(\neg y \Rightarrow a_1 \Rightarrow a_2 \Rightarrow \neg z\)
- \(\neg y \Rightarrow b_1 \Rightarrow \neg x\)
- \(y \Rightarrow \neg a_1 \Rightarrow \neg a_2 \Rightarrow z\)
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A colouring is possible in all cases.
Equality: \((x = x' \land y = y')\)
Equality: \((x = x' \land y = y')\)

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Equality: \((x = x' \land y = y')\)
Equality: \( x = x' \land y = y' \)
Equality: \((x = x' \land y = y')\)

\[
\begin{array}{c|ccc|c}
\hline
y & y_1 & y_2 & y_3 & y' \\
\hline
1,1 & 1,2 & 1,1 & 1,2 & 1,1 \\
1,1 & 1,2 & 1,1 & 1,2 & 2,2 \\
1,1 & 1,2 & 1,1 & 1,3 & 3,3 \\
1,1 & 1,2 & 1,1 & 1,4 & 4,4 \\
\hline
\end{array}
\]
Equality: \((x = x' \land y = y')\)

\[
\begin{array}{cccccc}
 y & y_1 & y_2 & y_3 & y' \\
 1,1 & 1,2 & 1,1 & 1,2 & 1,1 \\
 1,1 & 1,2 & 1,1 & 1,2 & 2,2 \\
 1,1 & 1,2 & 1,1 & 1,3 & 3,3 \\
 1,1 & 1,2 & 1,1 & 1,4 & 4,4 \\
 1,2 & 1,1 & 1,2 & 1,1 & 1,2 \\
 1,2 & 1,1 & 1,2 & 1,1 & 1,3 \\
 1,2 & 1,1 & 1,2 & 1,1 & 1,4 \\
 1,2 & & & & \\
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\end{array}
\]
Equality: \((x = x' \land y = y')\)
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Equality \((x = y = z)\)
Equality \((x = y = z)\)

\[x = y = z\]

\[x = x' \text{ and } y = y'\]
Equality \((x = y = z)\)

\[
x = y = z
\]

\[
x = x' \text{ and } y = y'
\]
Equality \((x = y = z)\)
More Simple Components

Weak Or:

\[ \neg x \land \neg z \implies \neg y \]
More Simple Components

Weak Or:
\[ \neg x \land \neg z \Rightarrow \neg y \]

Diagram:
```
  x'  y  y'  z'
    z  x   z
    y' x' y z'
```

Weak Negation:

\[ \neg x \Rightarrow y \] and \[ \neg y \Rightarrow x \]

True:
\[ x = \text{true} \]

Diagram:
```
  a  b  c
  1
```
More Simple Components

Weak Or:
\[ \neg x \land \neg z \Rightarrow \neg y \]

Weak Negation:
\[ \neg x \Rightarrow y \quad \text{and} \quad \neg y \Rightarrow x \]
More Simple Components

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True:
\[ x = \text{true} \]
More Simple Components

Weak Or:
\[ \neg x \land \neg z \Rightarrow \neg y \]

Weak Negation:
\[
\neg x \Rightarrow y \quad \text{and} \quad \neg y \Rightarrow x
\]

True:
\[ x = \text{true} \]
More Simple Components

Weak Or:
\[ \neg x \land \neg z \Rightarrow \neg y \]

Weak Negation:
\[ \neg x \Rightarrow y \text{ and } \neg y \Rightarrow x \]

True:
\[ x = true \]
Or \( x \lor y = z \)
Or \( (x \lor y = z) \)

\[
\begin{align*}
\neg x \land \neg y &\implies \neg x_3 \land \neg y_1
\end{align*}
\]
Or \((x \lor y = z)\)

\[
x \lor y = z
\]

A colouring is possible in all cases.
Or \((x ∨ y = z)\)

\[
\begin{align*}
\text{\(\lnot x \land \lnot y \Rightarrow \lnot x_3 \land \lnot y_1 \Rightarrow \lnot z_3 \Rightarrow \lnot z\)}
\end{align*}
\]
Or \((x \lor y = z)\)

\[\neg x \land \neg y \Rightarrow \neg x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z\]

\(x\)
Or \((x \lor y = z)\)

- \(x \land \neg y \Rightarrow \neg x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z\)
- \(x \Rightarrow \neg x'\)
Or $(x \lor y = z)$

- $\neg x \land \neg y \Rightarrow \neg x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z$
- $x \Rightarrow \neg x' \Rightarrow z_1$
Or \( (x \lor y = z) \)

- \( \neg x \land \neg y \Rightarrow \neg x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z \)
- \( x \Rightarrow \neg x' \Rightarrow z_1 \Rightarrow z \)
Or \((x \lor y = z)\)

- \(\neg x \land \neg y \Rightarrow \neg x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z\)
- \(x \Rightarrow \neg x' \Rightarrow z_1 \Rightarrow z\)
- \(y\)
Or \((x \lor y = z)\)

- \(\neg x \land \neg y \Rightarrow \neg x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z\)
- \(x \Rightarrow \neg x' \Rightarrow z_1 \Rightarrow z\)
- \(y \Rightarrow \neg y'\)
Or \((x \lor y = z)\)

- \(\neg x \land \neg y \Rightarrow \neg x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z\)
- \(x \Rightarrow \neg x' \Rightarrow z_1 \Rightarrow z\)
- \(y \Rightarrow \neg y' \Rightarrow z_4\)
Or \((x \lor y = z)\)

- \(\neg x \land \neg y \Rightarrow \neg x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z\)
- \(x \Rightarrow \neg x' \Rightarrow z_1 \Rightarrow z\)
- \(y \Rightarrow \neg y' \Rightarrow z_4 \Rightarrow z\)

A colouring is possible in all cases.
Or \((x \lor y = z)\)

- \(\neg x \land \neg y \Rightarrow \neg x_3 \land \neg y_1 \Rightarrow \neg z_3 \Rightarrow \neg z\)
- \(x \Rightarrow \neg x' \Rightarrow z_1 \Rightarrow z\)
- \(y \Rightarrow \neg y' \Rightarrow z_4 \Rightarrow z\)
- A colouring is possible in all cases.
$x_2 = x'_2$ and $(x_1 \oplus e_1) \lor (x_2 \oplus e_2) \lor (x_3 \oplus e_3) = true$
Static Simple Clause

\[ x_2 = x'_2 \text{ and } (x_1 \oplus e_1) \lor (x_2 \oplus e_2) \lor (x_3 \oplus e_3) = \text{true} \]
Multiple Equality ($x_i = y_i$)
Multiple Equality \((x_i = y_i)\)
Multiple Equality ($x_i = y_i$) [with Transport ($z_0 = z_k$)]
Multiple Equality \((x_i = y_i)\) [with Transport \((z_0 = z_k)\)]
Clause \((x_i = y_i \text{ und } c_i \text{ satisfied})\)
Clause \((x_i = y_i \text{ und } c_i \text{ satisfied})\)
Formula (all $c_i$ are satisfied)
Theorem

The $k$-colouring problem on circle-graphs is NP-complete for $k \geq 4$.

Theorem

The $(2 \cdot k - 1)$-colouring problem on circle-graphs with clique size $k$ is NP-complete for $k \geq 3$.

Theorem

A circle-graph with clique size $k$ is always $(3 \cdot k)$-colourable.
Indepedent Set and Clique

**Theorem**

*Finding a maximal independent set is solvable in time $O(n \log(n))$ on circle-graphs.*
Theorem

Finding a maximal independent set is solvable in time $O(n \log(n))$ on circle-graphs.

Theorem

Finding a maximal clique is solvable in time $O(n \log(n))$ on circle-graphs.
Concluding Remarks

Theorem

On an interval graph $G$ we may in time $O(n \log(n))$ compute $\chi(G), \alpha(G)$ and $\omega(G)$. 
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On an interval graph $G$ we may in time $O(n \log(n))$ compute $\chi(G)$, $\alpha(G)$ and $\omega(G)$.

Theorem

On a permutation graph $G$ we may in time $O(n \log(n))$ compute $\chi(G)$, $\alpha(G)$ and $\omega(G)$.
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The $k$-colouring problem on arc-graphs is solvable in polynomial time, but the colouring problem for arc-graphs is NP-complete.
Concluding Remarks

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On an interval graph $G$ we may in time $O(n \log(n))$ compute $\chi(G)$, $\alpha(G)$ and $\omega(G)$.

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Theorem

The $k$-colouring problem on arc-graphs is solvable in polynomial time, but the colouring problem for arc-graphs is NP-complete.

Theorem

The 3-colouring on circle-graphs is solvable in time $O(n \log(n))$. The $k$-colouring problem on circle-graphs is NP-complete for $k \geq 4$. 
Conclusions

- Colouring (and many more problems) on interval graphs are easy.
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- $k$-colouring on arc-graphs is easy.
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Conclusions

• Colouring (and many more problems) on interval graphs are easy.

• $k$-colouring on arc-graphs is easy.

• Colouring on arc-graphs is hard.

• 4-colouring problem on circle-graphs is hard.

• 3-colouring problem on circle-graphs is easy.

• $k$-colouring problem on circle-graphs is hard for $k > 3$. 
**g-Segment-graphs**

**Definition (g-Segment-graphs)**

- A graph $G = (V, E)$ is called $g$-Segment-graph, iff
Definition (g-Segment-graphs)

- A graph $G = (V, E)$ is called $g$-Segment-graph, iff
- it is the intersection-graph of a set of chords within a regular $g$-polygon.
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**Lemma**

*We have:*

1. A permutation-graph is a circle-graph.
**g-Segment-graphs**

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**Lemma**

*We have:*

1. A permutation-graph is a circle-graph.
2. A permutation-graph is a $g$-segment-graph.
g-Segment-graphs

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We have:

1. A permutation-graph is a circle-graph.
2. A permutation-graph is a $g$-segment-graph.
3. A proper arc-graph is a circle-graph.
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**Lemma**

We have:

1. A permutation-graph is a circle-graph.
2. A permutation-graph is a $g$-segment-graph.
3. A proper arc-graph is a circle-graph.
4. There are proper arc-graphs, which are not $g$-segment-graphs.
Definition (Disk-graphs)

A graph $G = (V, E)$ is called disk-graph, iff
Disk-graphs

Definition (Disk-graphs)

- A graph $G = (V, E)$ is called disk-graph, iff
- it is a intersection-graph of a set of disks in the plane.
Disk-graphs

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Definition (Disk-graphs)

- A graph \( G = (V, E) \) is called disk-graph, iff
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Definition (Unit-Disk-graphs)

- A graph \( G = (V, E) \) is called unit-disk-graph, iff
Disk-graphs

Definition (Disk-graphs)

- A graph $G = (V, E)$ is called disk-graph, iff
- it is a intersection-graph of a set of disks in the plane.

Definition (Unit-Disk-graphs)

- A graph $G = (V, E)$ is called unit-disk-graph, iff
- it is a intersection-graph of a set of equally sized disks in the plane.
**Disk-graphs**

**Definition (Disk-graphs)**

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Disk-graphs

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